BHUBANANANDA ODISHA SCHOOL OF ENGINEERING, CUTTACK
DEPARTMENT OF MECHANICAL ENGINEERING

LECTURE NOTE OF ENGINEERING MECHANICS

SUBJECT: ENGINEERING MECHANICS

FACULTY: ACHYUT KUMAR PATRA

ACCADEMIC SESSION: 2021-22

SEMESTER:2ND

Engineering Mechanics (CHAPTER-1)

Defn:
→ Mechanics is the Branch of physics which deals with the affect of force or multiple forces acting on a body or a system of body, which is at rest or in motion.

→ It deals with the "Macroscopic Approach". i.e the external effect of the body (velocity, displacement, acceleration etc.).

Types of Body:

(i) Rigid Body:
→ Rigid body is a body in which the distance between all the particles remains constant or in other words the deformation is zero, under the action of force.

(ii) Deformable Body:
→ It deforms easily under the action of force.
e.g rubber.

(iii) Fluid Body:-
→ fluid is something that can flow.
e.g liquid and gas.

* Note)
Rigid Body Idealization:
In Engineering mechanics we Idealize/Assume that in rigid body there is negligible deformation (no deformation) takes place under the action of external force, in order to simplify our studies.

Classification of Engineering Mechanics:

Mechanics

- Types of body
  - (i) Rigid body mechanics
  - (ii) Deformable body mechanics
  - (iii) Fluid body mechanics
- Types of motion
  - Statics (No motion)
  - Dynamics (in motion)
    - Kinematics
    - Kinetics

Statics:

→ Statics is the Branch of mechanics which deals with the body which is at rest.

→ It deals with the forces and their effects while acting upon a body is at rest.

* In Static mechanics the objective is to findout the unknown forces / Reaction forces of a body when it is subjected to any external force. This study helps us to findout the Internal effect acting on a body further.

Dynamics

→ Dynamics is the branch of mechanics which deals with the body which is in motion.

→ Dynamics deals with the forces and their effects while acting upon a body is in motion.

Kinematics: (without considering the forces)

→ It is the branch of dynamics which deals with the body in motion without any reference of the forces which are responsible for the motion.

Kinetics: (considering the forces)

→ It is the branch of dynamics which deals with the body in motion with the reference of the forces which are responsible for the motion.

Assumptions or Idealization:

(i) Particle:

→ In Engineering mechanics we assume that the entire body represented/replaced as a particle & the particle lies in the center of the body i.e center of mass.

→ "Particle" located at the center of the mass.

(ii) Rigid Body:

→ No deformation occures with the affect of forces.

(iii) Point load:

→ we replaced the entire load of a body into a single load, which acts from the center of the mass, Because we assume that the entire body as a particle.

FORCE

→ Force is defined as an agent which produces or tends to produce, destroys or tends to destroy the motion of the body whether the body is in rest condition or in motion."

→ Force is a vector quantity, i.e it has some magnitude as well as direction. It is represented as $\vec{F}$. Its unit is Newton (N)

Effect of a Force:

A force may produce following effect in a body:

(i) It may change the motion of a body.

(ii) It may retard the motion of a body.

(iii) It may retard the forces already acting on a body, thus bringing it to rest or in equilibrium.

(iv) It may increase the effect of the forces.

(v) It may give rise to the internal stresses in a body, on which it acts.

Characteristics of a force:

(i) Mangitude (10 N, 15 N, 20 N etc.)

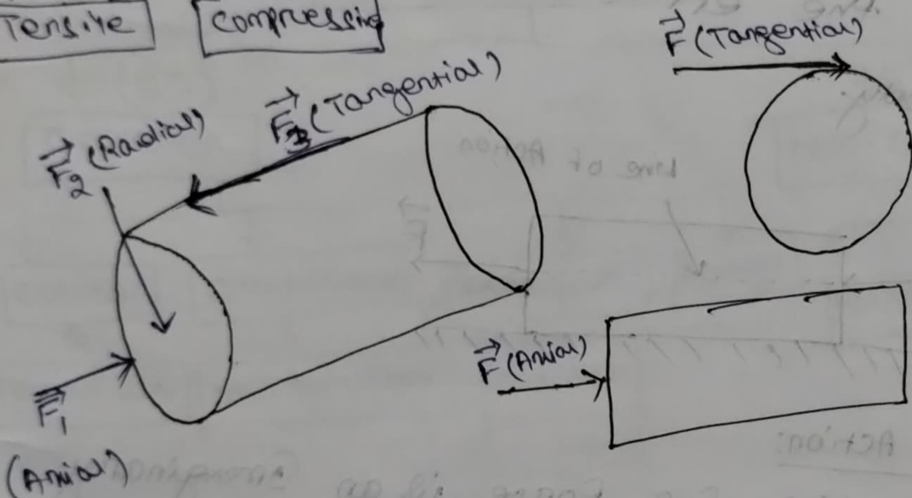
(ii) Direction (vertical, Horizontal, Angle, sense) i.e right / left or up & down

(iii) Nature of the force (Push or Pull)

(iv) Point of action/Application (The point at which or through which the force acts on a body)

Types of Force:

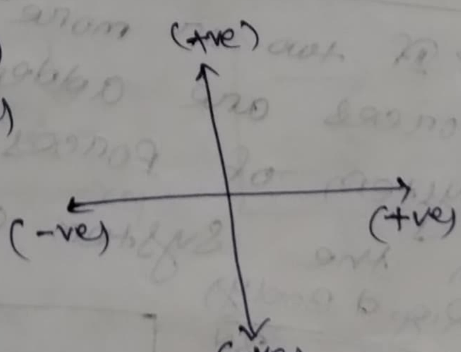
- Force
  - Applied Type
    - Axial
      - Tensile
      - Compressive
    - Radial
    - Tangential
  - Non-Applied Type
    - Frictional Force
    - Self weight / Gravitational Force
    - Reaction Force



* Except this catagorisation force may be also classified as (i) contact type &
(ii) non-contact type

Sign convention of force:

→ (+ve) ↑ (+ve)
← (-ve) ↓ (-ve)

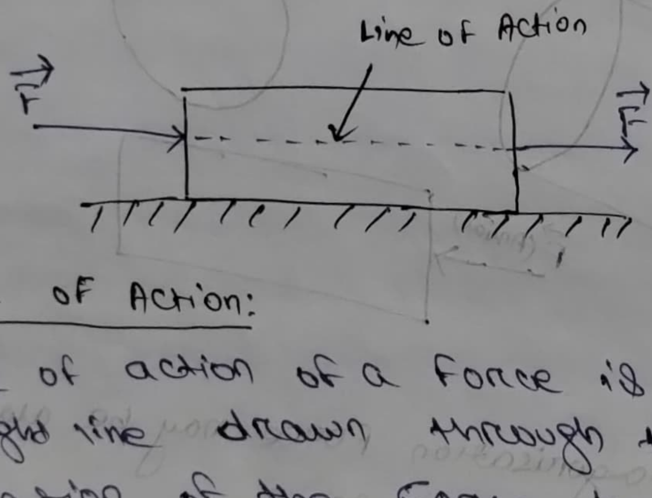


* Some other forces are Mechanical force, spring force, Equilibrium force, resultant force etc.

Principle of Transmissibility: 70

→ This principle states that " If a force acts at any point on a rigid body, It may also be considered to act any other points on its line of action, provided this point is rigidly connected with the body

→ In other words, principle of Transmissibility states that " The force can be represented anywhere along the line of action without changing the effect of the forces on any rigid body

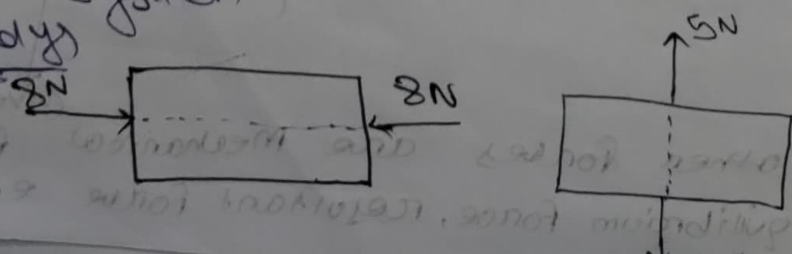


* **Line of Action:**

→ Line of action of a force is an Imaginary straight line drawn through the point of Application of the Force in the direction of applied force.

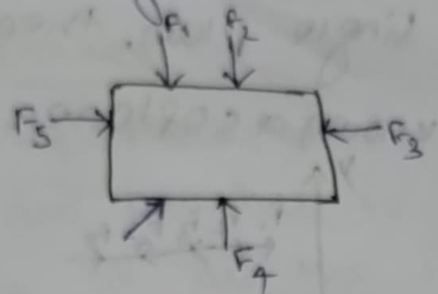
Principle of Superposition of Forces:

→ If two or more equal and opposite colinear Forces are added or removed from the system of forces, there will be no change in the system and position of the body. (Rigid Body)

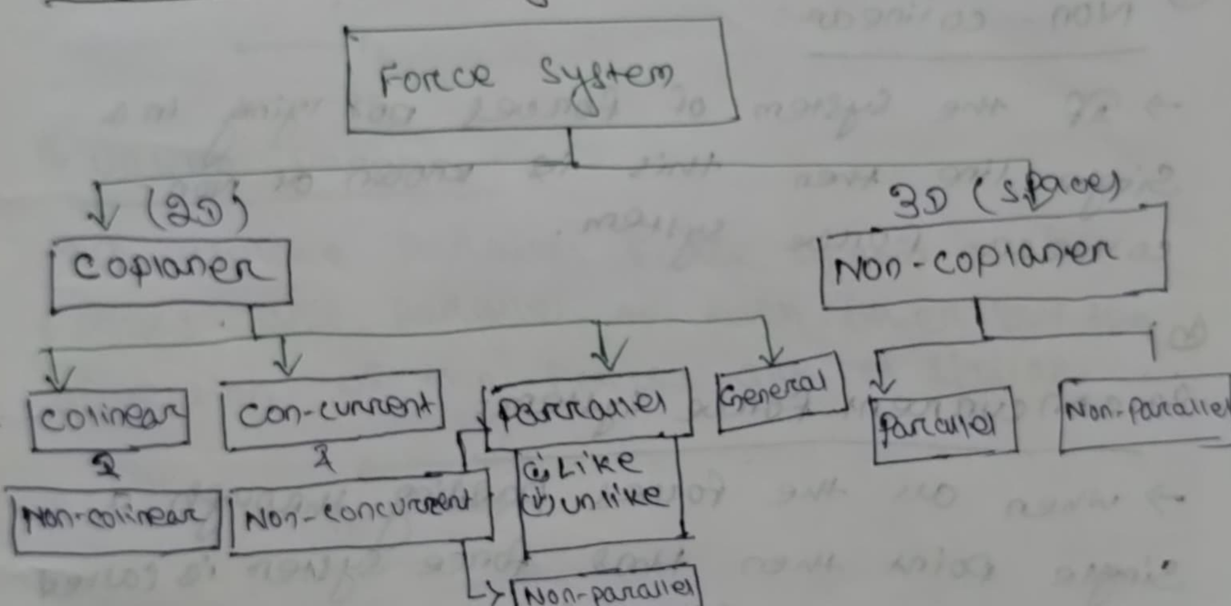


FORCE SYSTEM

→ When two or more forces acts on a body, they are called to form a system of forces or Force system.

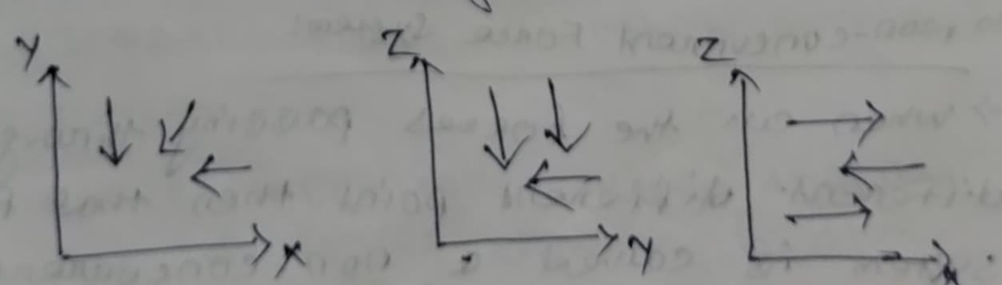


Types of Force system:



① Coplaner force system:

→ The forces, when acting in a single plane i.e (X-Y)/(Y-Z) or in (X-Z) plane are known as coplaner force system.



* Force direction may be in outward or in inwards towards the point.

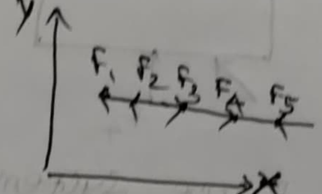
→ (outward from the point)

← Towards the point

(a)

i) coplaner colinear Force system:

→ If the forces are acting in a single straight line as well as all the forces are acting on a single plain, then this force system is known as coplaner colinear force system.



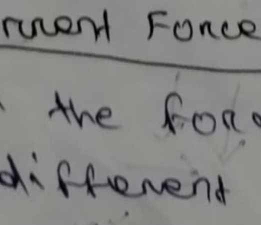
ii) Non - colinear:

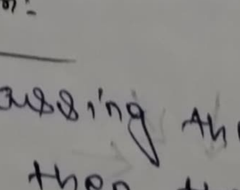
→ If the system of forces not lying in a single line then this is known as non-colinear force system.

(b)

i) concurrent force system:

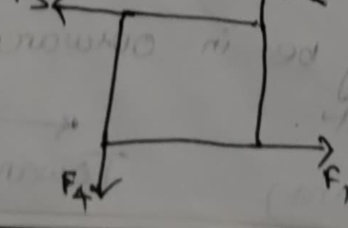
→ when all the forces passing through a single point then that force system is called a concurrent force system.





ii) non-concurrent Force system:

→ when all the forces passing through a different different point then that force system is called a non-concurrent force system.



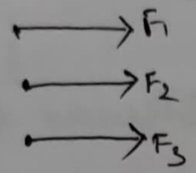
(c)

Parallel Force System:

→ In parallel force system all the forces are parallel to each other.

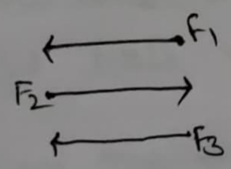
(i) Like parallel:

→ In like parallel force system all the forces are parallel to each other and the direction of all the forces are same.



(ii) Unlike parallel:

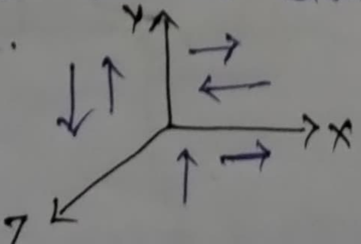
→ In unlike parallel force system all the forces are parallel to each other but the direction of the forces are not similar.



(2) Non-coplaner Force System:

→ When the system of forces are not acting on a single plane (i.e ~~...~~ acting on different planes i's called Noncoplaner force system.

→ In non-coplaner force system the line of action of forces donot lie in the same plane.



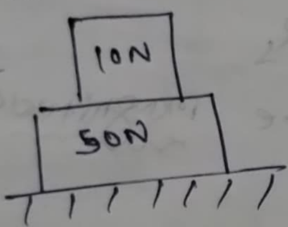
Free Body Diagram

→ Free Body Diagram (FBD), it simply means we have to free the body from the supports, contact surfaces and to draw a separate diagram in which we have to show the support reactions, surface reactions of removed support as well as the surfaces.

Case-1

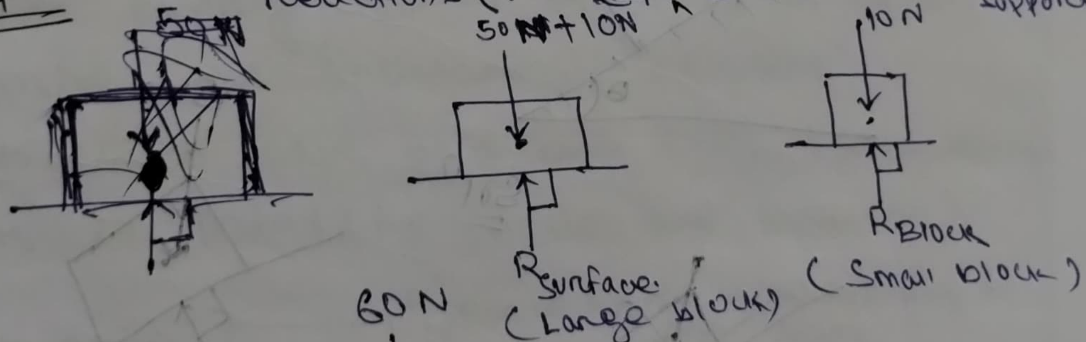
* Support reactions must be perpendicular to the supports.

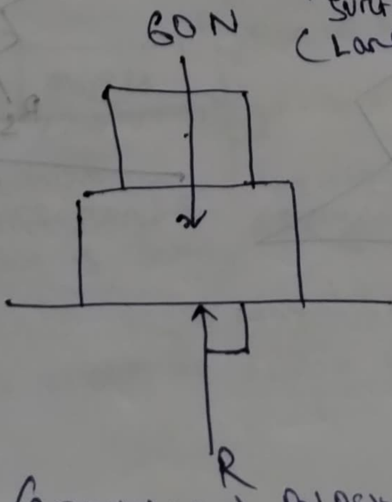
Q.



Step-1 → show the self weight of the body (Self weight must be centrally vertically downward direction)

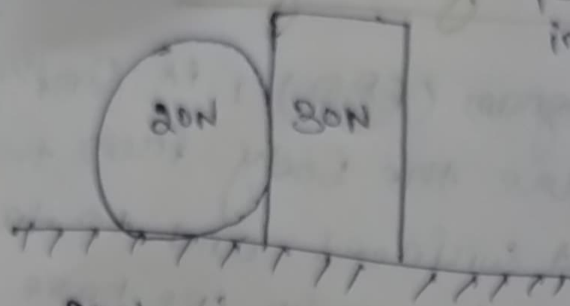
Step-2 → Show the surface reactions and support reactions (i.e perpendicular to the surface or supports)



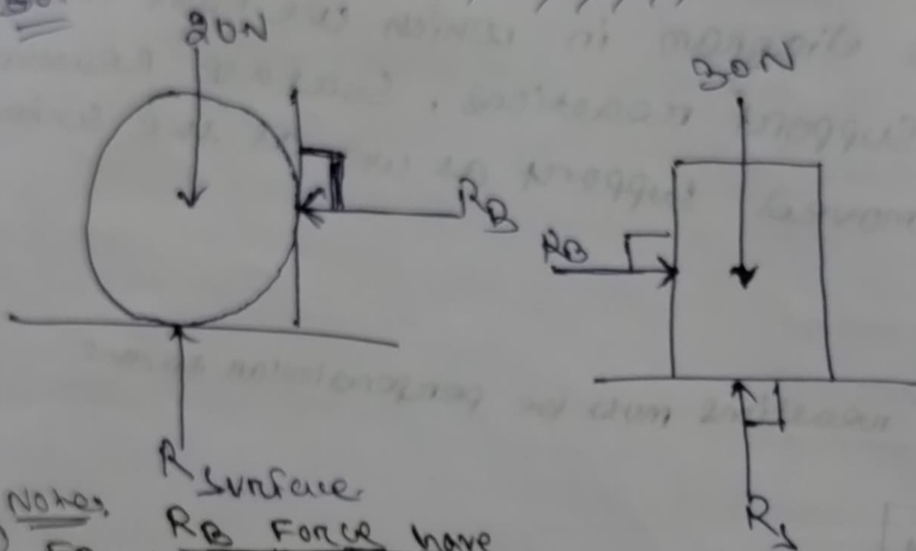


Note:
→ Self weight: [redacted]: centrally vertically downward.
→ Surface/Support reaction:
→ perpendicular to the surface or contact supports

case-2 (contact reaction) { contact reaction must be towards the center in case of cylinder / sphere

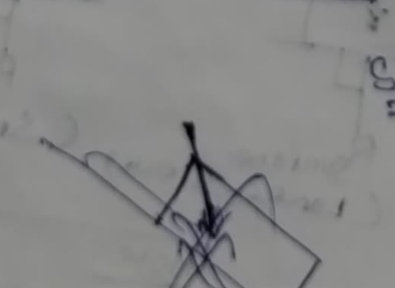


Soln

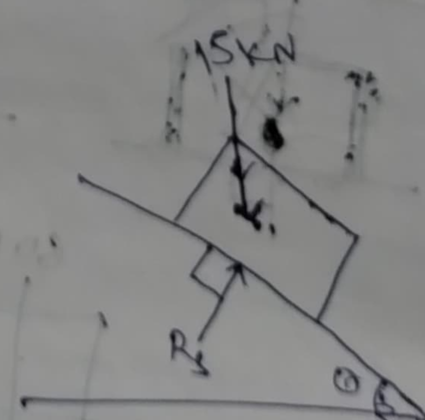


Notes: RB Force have Equal and opposite reaction i.e magnitude same, direction opposite.

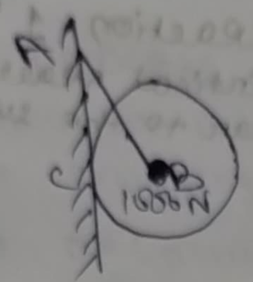
case-3



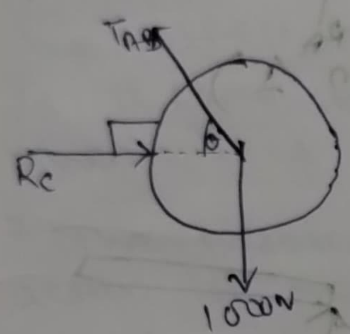
Soln



Case-4



* cord, cable, wire, rope, thread, String material we have to Show the tension.



~~Case-5~~

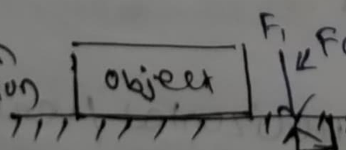
Action and Reaction forces.

This comes under the Newton's 3rd law of motion. which States that:

"For every action there is an equal, opposite and Instataneous reaction".

* No Single force on a body exits, Force always acts in pairs.(i.e Action and Reaction)

* Both Action Force & Reaction Force have Same magnitude but direction is different & point of Application is Same.



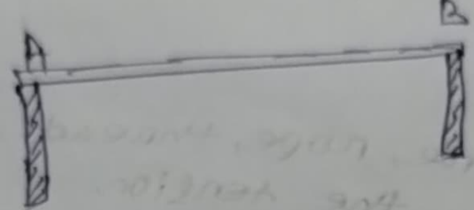
Force by the object Applying on the table.

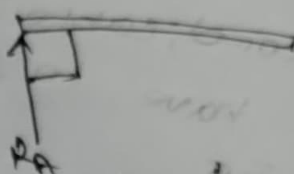
Reaction Force by the surface applying on the object

* The external force / Internal force which is applying on a body is known as action force and

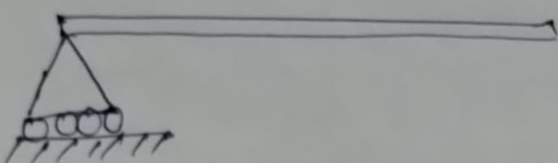
Types of supports role in F.B.D:

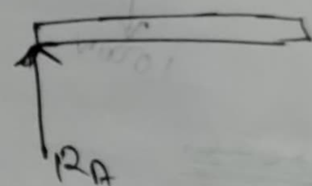
① **Simple support:** (Support reaction 1)
vertical reaction
perpendicular to the surface



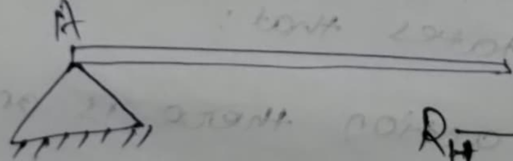


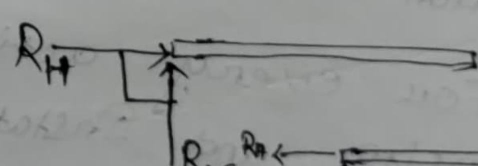
② **Roller support:** (Support reaction 1)



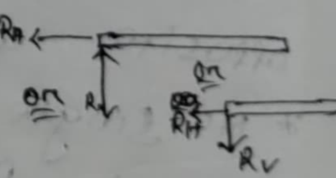


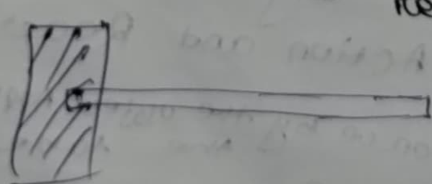
③ **Hinge support** (Support reaction 2)



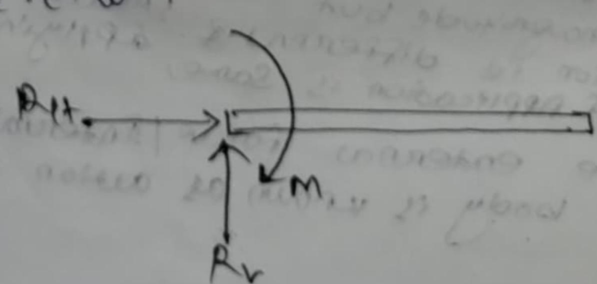


④ **Fixed support:** (Support reaction -2)





D.O.F : No travel in horizontal/vertical
c.w / c.c.w

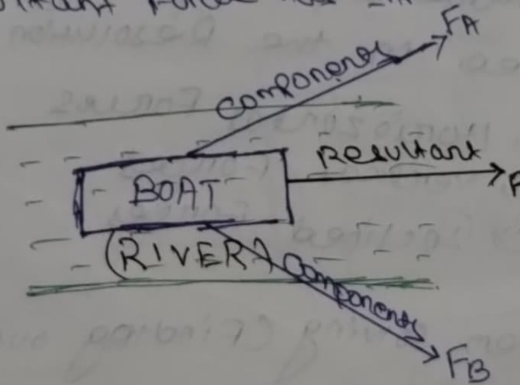


Resolution of a Force:

-> The process of splitting up the given force into two or a number of components/ direction without changing the effect on the body is called resolution of a Force.

-> In otherwords, when a single force acting on a body is replaced by two forces or multiple forces acting in such a way that the effect produced by this replaced forces is a same effect as that of single force. Then it is called resolution of a Force.

* The resolution of a force is the reverse action of the addition of the component vectors.
* The resultant force has infinite no. of components.



(*) Here the resultant Force $\vec{F}$ is separated by two components $\vec{F_A}$ & $\vec{F_B}$.

* These two forces ($\vec{F_A}$ & $\vec{F_B}$) are called components of a Force, and the force which is broken into two parts ($\vec{F}$) is called Resultant of the Resolved part.

Resultant Force:

-> If a number of forces are acting simultaneously on a particle, then it is possible to findout a single force which could replace them i.e which would produce the same effect as produced by the given forces. This single force is called "Resultant Force" and the given forces are called its components

Types of component Forces:

(i) perpendicular components:

(ii) Non-perpendicular components.

* Method of Resolution of a Force:

→ Method of Resolution of a force, simply means to findout the values of its various components of a single force (Resultant force).

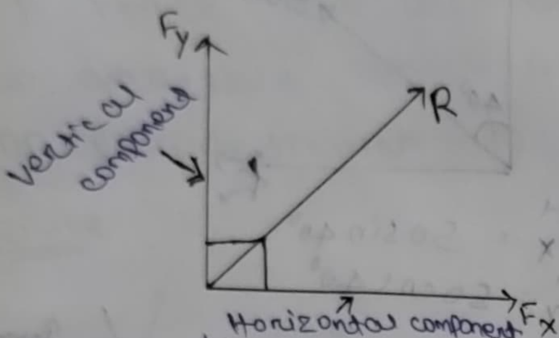
→ Generally there are two types of components as stated above i.e perpendicular and non-perpendicular components.

Resolution of a Force into two-Perpendicular components:

→ Generally there are three types of forces we have to see in the Resolution of forces, these are
(i) Horiozontal Forces
(ii) vertical forces
(iii) Inclined forces.

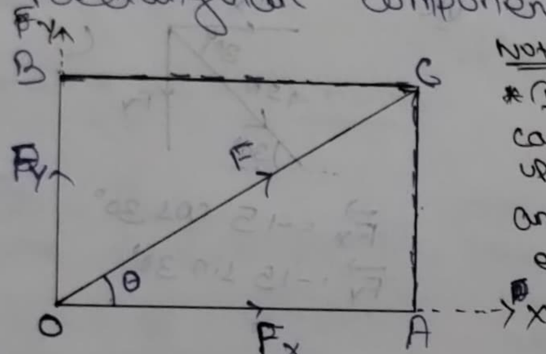
→ while problem solving (finding out the Resultant force) it is easy to observe or calculate the effect of the Horiozontal forces as well as the vertical forces, but it is very difficult to findout the effect of the Inclined forces because they are not acting on a certain angle and they acts in diffent angles. e.g: (30°, 40°, 50°, 70°, 120°, 150° etc)

→ In case of perpendicular component forces we have to split the resultant force into two components in such a way that they are (two components) (Horizontal component & vertical component) are perpendicular to each other.



* Here F_x and F_y are two perpendicular components of Resultant 'R'.

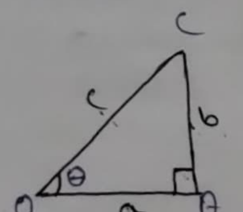
→ perpendicular components are also consided as the rectangular components.



Note:
* In the incline force calculation we are followed up by some inclination angle (Angle is known), either it is given directly or we have to findout this incline angle by the help of other given angle.

Derive

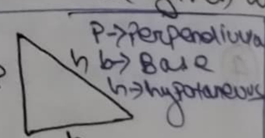
In △ OAC



$\sin\theta = \frac{p}{h}$

$\cos\theta = \frac{b}{h}$

$\tan\theta = \frac{p}{b}$



$\frac{a}{c} = \cos\theta$

$\Rightarrow \frac{\vec{F_x}}{\vec{F}} = \cos\theta$

$\Rightarrow \boxed{\vec{F_x} = \vec{F}\cos\theta}$

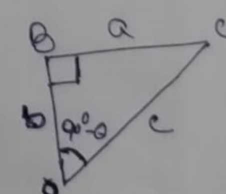
again

$\frac{b}{c} = \sin\theta$

$\Rightarrow \frac{\vec{F_y}}{\vec{F}} = \sin\theta \quad (\because \overline{AC} = \overline{OB} = \vec{F_y})$

$\Rightarrow \boxed{\vec{F_y} = \vec{F}\sin\theta}$

or In △ OBC



$\frac{b}{c} = \sin(90° - \theta)$

$\Rightarrow \frac{\vec{F_y}}{\vec{F}} = \cos\theta$

$\Rightarrow \boxed{\vec{F_y} = \vec{F}\cos\theta}$

again

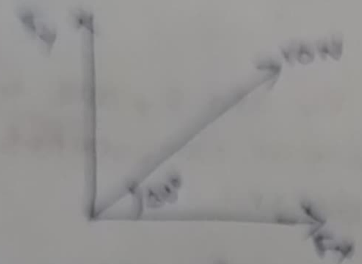
$\frac{a}{c} = \cos(90° - \theta)$

$\Rightarrow \frac{\vec{F_x}}{\vec{F}} = \sin\theta$

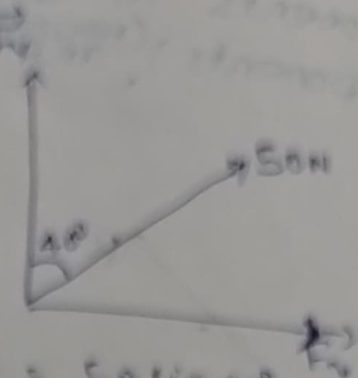
$\Rightarrow \boxed{\vec{F_x} = \vec{F}\sin\theta}$

Note.→ The "cos" component of force is always in the direction along axis from angle (θ) or given force.

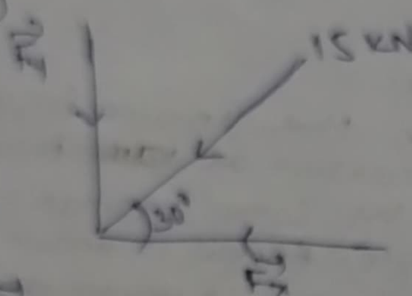
Problems

① 

$\vec{F_x} = 10\cos 30°$
$\vec{F_y} = 10\sin 30°$

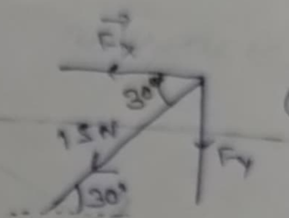
② 

$\vec{F_x} = 50\sin 40°$
$\vec{F_y} = 50\cos 40°$

③ 

$\vec{F_x} = -15\cos 30°$
$\vec{F_y} = -15\sin 30°$

or



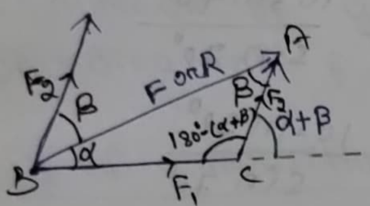
$\vec{F_x} = -15\cos 30°$
$\vec{F_y} = -15\sin 30°$

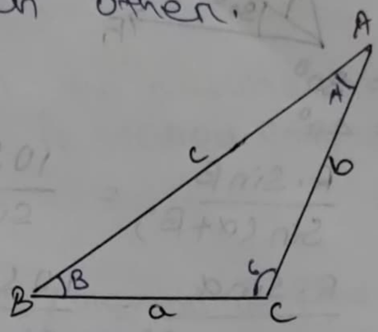
④

Resolution of a Force into two Non-perpendicular components

→ A force can also be resolved along the two directions which are not at right angle / non-perpendicular to each other.

Derive





So According to this rule:

→ In any △ ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where a, b and c are the lengths of the three sides of a triangle, A, B and C are opposite angles of the three angles of the sides a, b & c respectively.

$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\alpha} = \frac{R}{\sin[180^\circ-(\alpha+\beta)]}$$

$$\Rightarrow \boxed{\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\alpha} = \frac{R}{\sin(\alpha+\beta)}}$$

$(\because \sin(180^\circ-\theta) = \sin\theta)$

So

$$\frac{F_1}{\sin\beta} = \frac{R}{\sin(\alpha+\beta)}$$

$$\Rightarrow \boxed{\vec{F_1} = \frac{R\cdot\sin\beta}{\sin(\alpha+\beta)}}$$

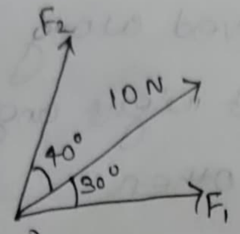
$$\frac{F_2}{\sin\alpha} = \frac{R}{\sin(\alpha+\beta)}$$

$$\Rightarrow \boxed{\vec{F_2} = \frac{R\cdot\sin\alpha}{\sin(\alpha+\beta)}}$$

So using this above two rules we can find the components of a force in two non-perpendicular directions.

Problems:

(1)



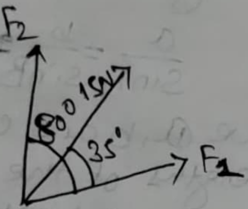
$\alpha = 30^\circ$

$\beta = 40^\circ$

$$F_1 = \frac{R \cdot \sin\beta}{\sin(\alpha+\beta)} = \frac{10\sin 40^\circ}{\sin(40+30)} = \frac{10\sin 40^\circ}{\sin 70^\circ}$$

$$F_2 = \frac{R \cdot \sin\alpha}{\sin(\alpha+\beta)} = \frac{10\sin 30^\circ}{\sin(40+30)} = \frac{10\sin 30^\circ}{\sin 70^\circ}$$

(2)



$\alpha = 35^\circ$

$\beta = 80^\circ - 35^\circ = 45^\circ$

$$F_1 = \frac{R\sin\beta}{\sin(\alpha+\beta)} = \frac{15\sin 45^\circ}{\sin(80)^\circ}$$

$$F_2 = \frac{R \cdot \sin\alpha}{\sin(\alpha+\beta)} = \frac{15\sin 35^\circ}{\sin 80^\circ}$$

COMPOSITION OF FORCES

Resultant Force:

As we discussed earier, Resultant Force is a single force which produces the same effect as a number of forces acting together.

Composition of Forces:

→ The process of finding out the resultant force, of a number of given force is called composition/compounding of forces.

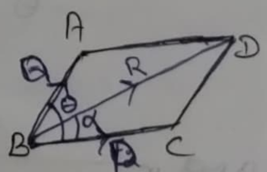
Method of composition of forces / method of finding Resultant force:

(i) Analytical method
(ii) Graphical method

Analytical Method For Finding Resultant Force:

→ The resultant force, of a given system of forces, may be found out analytically by the following methods:

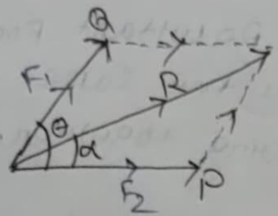
(i) parallelogram law of forces
(ii) Method of Resolution



Parallelogram Law of Forces

→ In a parallelogram opposite sides are parraller to each other.

→ Parallelogram Law of Forces states that "If two ~~[illegible]~~ forces, acting simultaneously on a particle, be represented in Magnitude and direction by the two adjacent sides of a parallelogram; there resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of Intersection."



Note:
Whenever there are two forces acting on a plane intersecting each other, we use "Parallelogram" law of forces for finding out of "Resultant".

* This principle applies where,

(i) There are two concurrent forces.
(ii) These forces have same (direction)

Mathematically,

Resultant force

$$\boxed{R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} \quad \text{or} \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

where,
P and Q = Two concurrent forces acting on a single plain

θ = It is the angle between these two forces P & Q.

and

$$\boxed{\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}} \quad \text{or} \quad \boxed{\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}}$$

F_1 and F_2 → Forces whose resultant is required to be found out.

θ → Angle between the forces $\vec{F_1}$ & $\vec{F_2}$

α → Angle which the resultant force makes with one of the forces ($\vec{F_2}$ or $\vec{F_1}$)

$$\boxed{\alpha = \tan^{-1}\left|\frac{F_2\sin\theta}{F_1 + F_2\cos\theta}\right|} \quad \text{or} \quad \boxed{\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)}$$

problems:

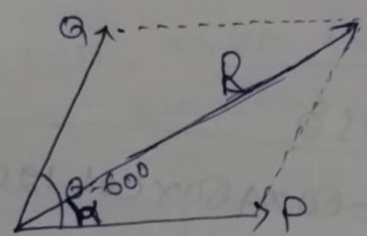
Q. Calculate the resultant of two forces having magnitude 600N and 450 N and acting at 60° to one another, when they have

① same sense
② opposite sense

Solve

① Same sense:

P = 600N, Q = 450N, θ = 60°



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{(600)^2 + (450)^2 + 2\times600\times450\cos60^\circ}$$

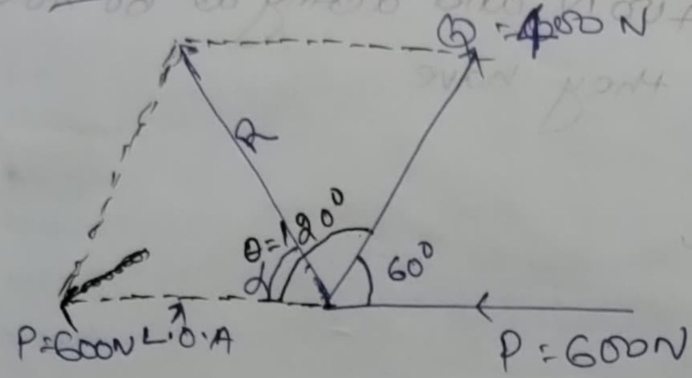
$$\boxed{R = 921.773\ N}$$

$$\alpha = \tan^{-1}\left|\frac{Q\sin\theta}{P + Q\cos\theta}\right|$$

$$= \tan^{-1}\left|\frac{450\sin60^\circ}{600 + 450\cos60^\circ}\right|$$

$$\boxed{\alpha = 23.413^\circ}$$

(2) Opposite Sense



$P = 600N$
$Q = 400N$
$\theta = 120^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$
$$= \sqrt{(600)^2 + (400)^2 + 2\times 600\times 400\times \cos 120^\circ}$$
$$= 529.15 N$$

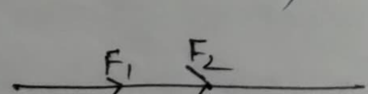
$$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$
$$= \tan^{-1}\left(\frac{400\sin 120}{600 + 400\cos 120}\right)$$
$$= 40.89^\circ$$

Note: Case-1

(1) If $\theta = 0$ (When the forces act along the same line

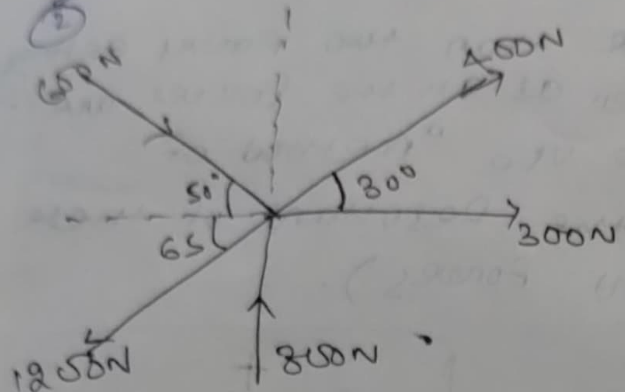
$$\boxed{R = F_1 + F_2}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$
$$= \sqrt{P^2 + Q^2 + 2PQ} \quad (\because \cos 0^\circ = 1)$$
$$= \sqrt{(P + Q)^2}$$

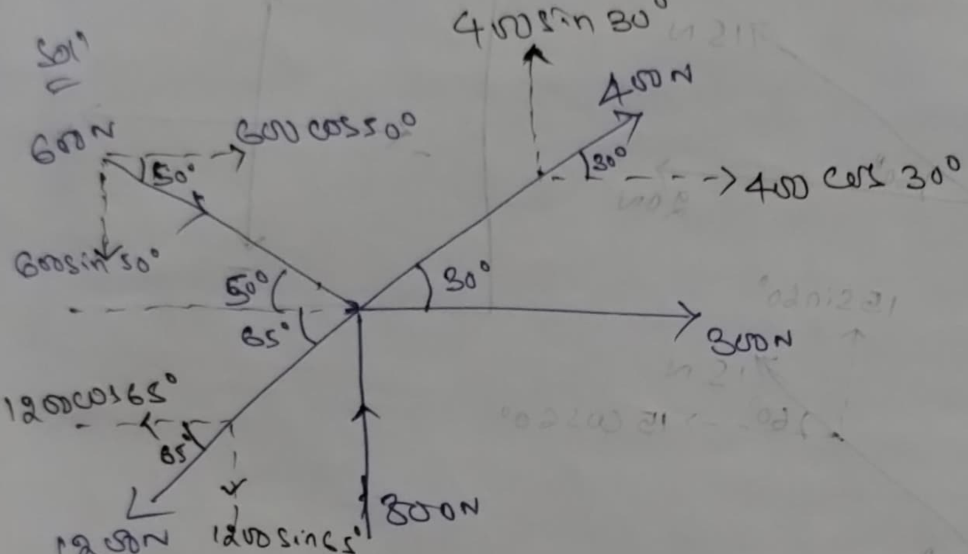


So, $\boxed{R = P + Q}$

Q. 2



Sol:



$\Sigma F_x = 300N + 400\cos 30^\circ + 600\cos 50^\circ - 1200\cos 65^\circ$
$= 524.94N$

$\Sigma F_y = 800 + 400\sin 30^\circ - 600\sin 50^\circ - 1200\sin 65^\circ$
$= -547.196N$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(524.94)^2 + (-547.196)^2}$

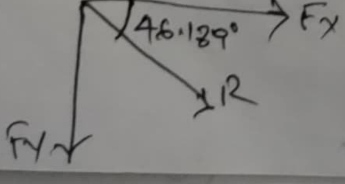
$\boxed{R = 758.277N}$

$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{-547.196}{524.94}\right) =$

$\boxed{\theta = 46.189^\circ}$

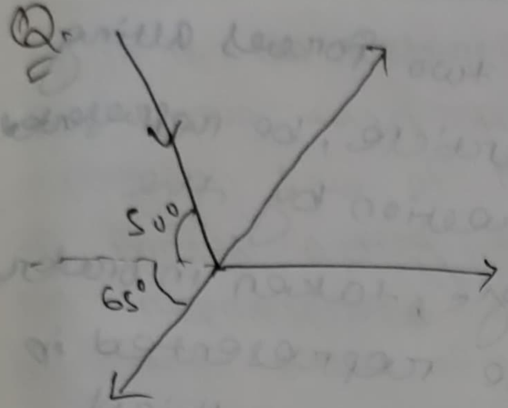
Note: θ will always kept in +ve sign. no need to write it in -ve sign.

→ The resultant lies 4th quadrant (x → +ve, y → -ve)

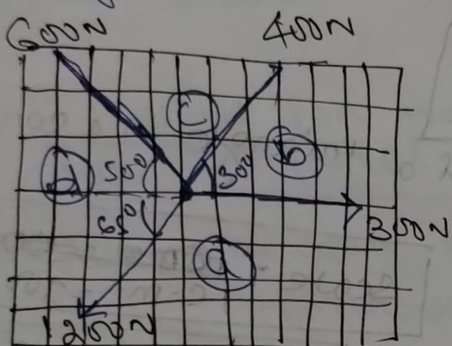


Graphical method

Q.



→ First, we have to draw space diagram. Exact angle and exact / Actual direction in the graph page.



→ sectioning it in C.W or in C.C.W direction. by using bow's notation.

→ In the vector diagram starting from a convenient point and then go on adding all the forces vectorically one by one (keeping in view the direction of the forces) to some suitable scale.

→ The graphical method for the resultant force is briefly discussed below.
There is two method (Graphically)

(i) Triangle Law of forces (Two forces)
(ii) polygon law of forces. (More than two forces)

GRAPHICAL METHOD

Triangle Law of Forces:

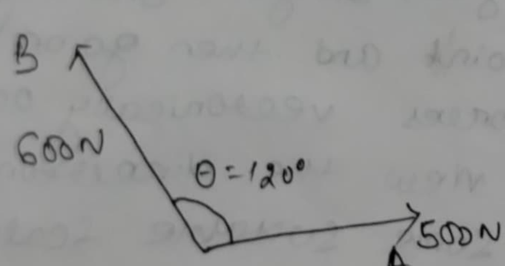
→ It states that "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order, there resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.

$$\boxed{\vec{A} + \vec{B} = \vec{R}}$$

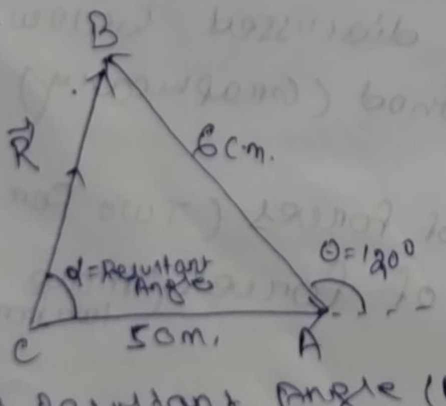
* Forming two sides of a triangle in a contineous line form.

Scale : ~~10cm = 100N~~ 1 c.m. = 100 N

eg



Sol^n



A

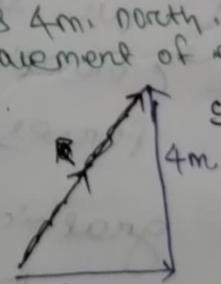
$\alpha \to$ Resultant Angle (Angle Inclined to Horizontal side)

$\vec{CB} \to$ magnitude of Resultant (Third side of the triangle taken in opposite order)

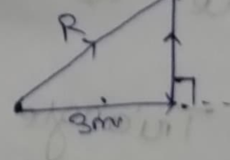
Try to Solve

Q. A girl moves 3m towards east, then she takes 90° left turn and moves 4m. north. Find displacement of girl.

Scale: 1 cm = 1m



Ans.



Polygon Law of Forces:

→ It is an extension of "Triangle Law of Forces" for more than two forces.

→ Polygon Law of forces states that "If a number of forces acting simultaneously on a particle be represented in magnitude and direction, by the sides of a polygon taken in order, then the resultant of all these forces may be represented in magnitude and direction by the closing side of the polygon, taken in opposite order.

Graphical method for the Resultant Force by polygon law of forces:

① Construction of space diagram (position Diagram):

→ It means the construction of a diagram showing the various along with their magnitude and lines of action.

② Use of Bow's notation:

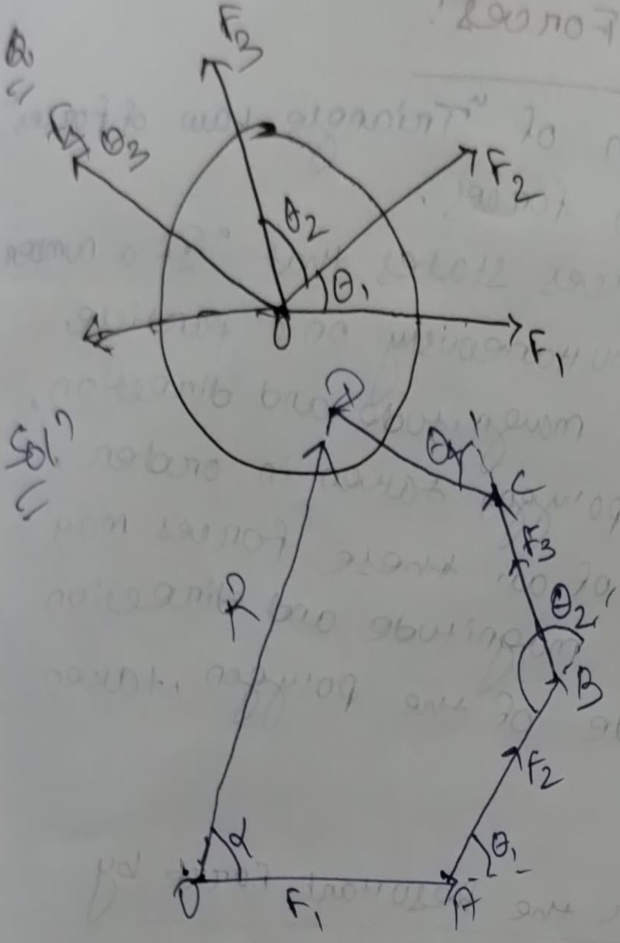
→ All the ~~methods~~ forces in the space diagram are named by using the Bow's notation. It is a convenient method in which every force is named by two capital letters, placed on its either sides in the space diagram.

③ Construction of vector diagram (Force diagram):

→ It means the construction of a diagram starting from a convenient point and then go on adding all the forces vectorically one by one (keeping in view the direction of the forces) to some suitable scale.

* Now the closing side of the polygon, taken in opposite order will give the magnitude of the resultant force (to the scale) and its direction.

Q
4



Q.
= (i) 20N at 30° North of east
(ii) 25N towards North
(iii) 30N towards N.W.
(iv) 35N at 40° towards

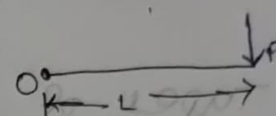
Moment of Force

Defⁿ:

→ It is the turning effect produced by a force on the body, on which it acts, about a point.

→ The ability of a force to produce rotational motion. or the rotational effect of a force.

→ It is also known as Torque.



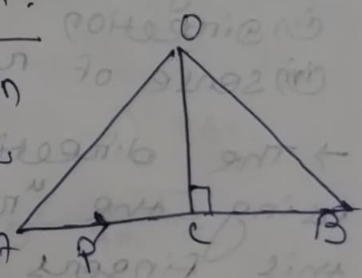
→ moment depend upon two factors :

$\boxed{\text{Torque/moment} = \text{Force} \times \text{moment arm}}$ → $\boxed{M = F \times L}$

moment arm = perpendicular distance between the line of action of force and the axis of rotation.

$$\boxed{\text{moment} = \text{Force} \times \text{perpendicular distance}}$$

Graphical representation of moment :



→ consider a force P represented, in magnitude and direction, by the Line AB.

→ Let O be a point, about which the moment of this force is required to be found out.

→ So in order to findout this moment we have to draw a perpendicular line to AB from the point O.

Now moment of force P about O point

$= P \times OC$

or

$AB \times OC$

$= 2 \times \frac{1}{2} \times AB \times OC$

$= 2 \times \text{Area of } \triangle OAB$

→ Hence moment of a force about a point is equal to twice of the Area of the triangle.

Units of moment:

Since the moment of a force is the product of force and distance, so the unit of moment will be, $\boxed{\text{(i) N-m}}$ → S.I unit
(ii) KN.m
(iii) N.mm etc

$$\boxed{1 \text{ KN.m} = 10^6 \text{ N.mm}}$$

Types of moment:

Two types
(i) Clockwise moment (C.W) (-ve)
(ii) Anticlockwise moment (C.C.W) (+ve)

Note:
→ Moment of force is a vector quantity, so it has
(i) magnitude (Force × ⊥r distance)
(ii) direction
(iii) sense of rotation.

→ The direction of moment will be obtained by using the "right hand thumb" rule. To do this fingers of the right hand are folded such that they follow the sense of rotation. The thumb then points along the moment axis which gives the direction and sense of the moment.

Principle of moment:

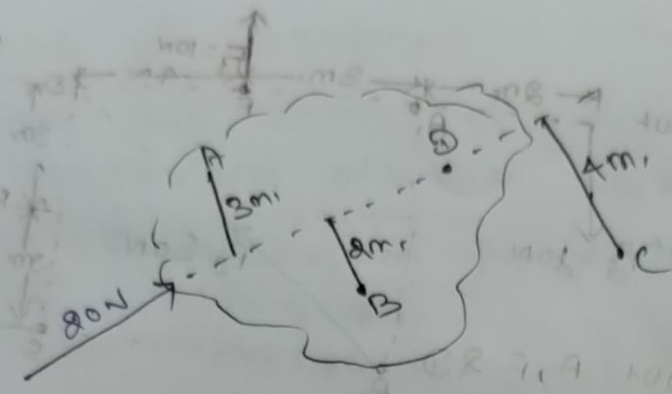
If a body is in equilibrium under the action of number of forces acting simultaneously on the body, then the algebric sum of the clockwise moments of the forces about a point is equal to the sum of anticlockwise moments of the forces about the same point.

Mathematically,

$$\boxed{\Sigma M = 0}$$

The algebric sum of moments of all the forces about a point, in their plane is Zero

Q.



find

(i) $M_A = 20\times3 = +60\ N\text{-}m$ [(c.c.w)]

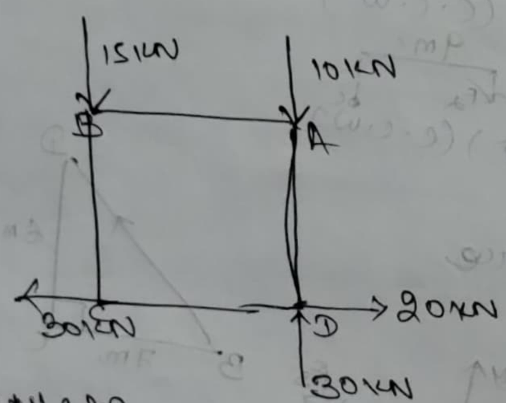
(ii) $M_B = 20\times2 = -40\ N\text{-}m$ [(c.w)]

(iii) $M_C = 20\times4 = -80\ N\text{-}m$ [(c.w)]

(iv) $M_D = 20\times0 = 0$

Sense of rotation

Q.



Calculate the moment at point B of various forces.

$\Sigma M_B =$

Varignon's Theorem:

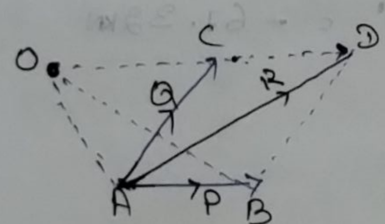
→ In a coplanar force system if a number of forces are acting on a body, then the algebraic sum of their moments about a point in their plane is equal to the moment of their resultant about the same point.

→ It is used to find the location of the point of application of resultant force w.r.t. any point or axis.

Sum of moment of Individual Forces = moment of resultant force alone

proof

$$\boxed{\sum M_O = R \times x}$$



Take a point O along the line of action of the Force $\vec{CD}$, such that away CD = OC

moment of Force $\vec{P}$, about point 'O'

= 2 × Area of △ AOB

moment of force $\vec{Q}$, about point 'O'

= 2 × Area of △ AOC

moment of Resultant $\vec{R}$, about point 'O'

= 2 × Area of △ AOD

△ AOD = △ AOC + △ ACD

⇒ △ AOD = △ AOC + △ AOB

We know,
△ ACD = △ ABD / 2
△ ABD = △ AOB
So,
△ ACD = △ AOB

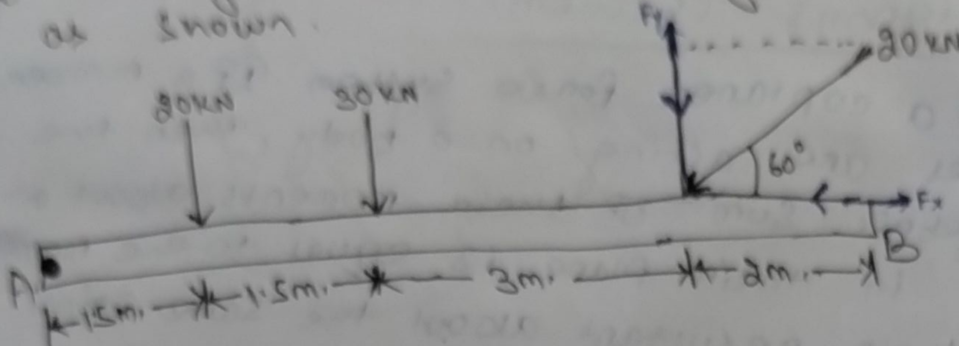
moment of Resultant force $\vec{R}$

$\vec{R}$ = 2 × Area of △ AOD

⇒ $\vec{R}$ = 2 × Area of (△ AOC + △ AOB)

proved

Q. The system of forces acting on a beam AB as shown.



Determine the magnitude, direction and location of point of application of resultant, in the given force system w.r.t. point A.

Sol^n

We can solve the Resultant force by "method of resolution".

$\Sigma H = -20\cos 60^\circ = -10KN$

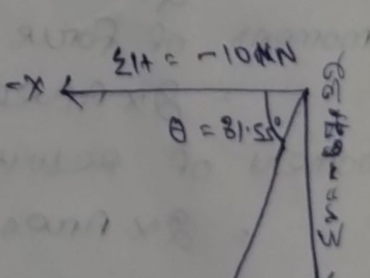
$\Sigma V = -20-30-20\sin 60^\circ = -67.32KN$

$R = \sqrt{(\Sigma H)^2+(\Sigma V)^2}$

$= \sqrt{(-10)^2+(-67.32)^2}$

$= 68.05 KN$

Both ΣH and ΣV in -ve sign, so the Resultant lies in 3rd quadrant.



$\theta = \tan^{-1}\left|\frac{\Sigma V}{\Sigma H}\right|$

$= \tan^{-1}\left|\frac{-67.32}{-10}\right|$

$= 81.55^\circ$

According to Varignon's Theorem,

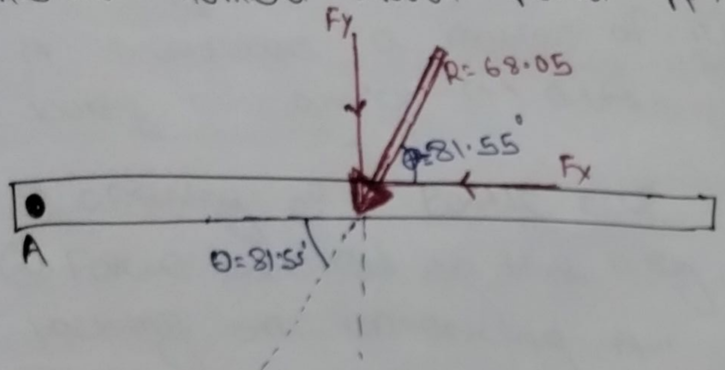
Sum of moment of Individual force = moment of Resultant force alone.

Here, $\boxed{\Sigma M_A = R \times x}$

Note: The resultant force lies in a inclined angle so that in calculation, we have to resolve this forces into two component. (The resultant force lies somewhere in the above beam)

$\sum M_A = -20\times 1.5 - 30\times 3 - 20\sin 60^\circ \times 6 + 0$ (Fx component)
$= -223.92 KN$

let's assume that the resultant force is acting on ~~any~~ point at a distance of x, due to the inclined angle of resultant force it also have two components and we have to calculate these components moment about point A.



$R\times x = \sum M_A$

$\Rightarrow \underbrace{-68.05\times \sin 81.55^\circ}_{\text{Fy component of Resultant R}} \times x = -223.92$

[$\vec{F_x}$ component line of action lies in the point A, so the moment at point A is zero for $\vec{F_x}$ component.]

$\Rightarrow x = \dfrac{-223.92}{-68.05\times \sin 81.55^\circ}$

$\Rightarrow x = 3.32$ m.

So the resultant lies at 3.32 m. distance from point A.