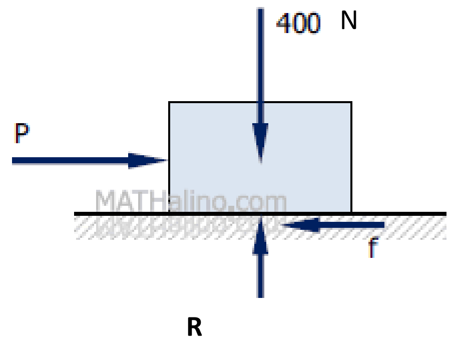
LEFTOUT PORTION OF CHAPTER 3

Q. A 400 N block is resting on a rough horizontal surface for which the coefficient of friction is 0.40. Determine the force P required to cause motion to impend if applied to the block horizontally.

Let the Force is applied horizontally is P.



ΣFV=0

⇨ R=400 N

Ff=μR=0.40(400)

⇨ Ff=160 N

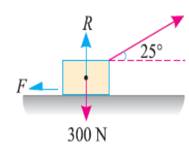
ΣFH=0

⇨ P=Ff

P=160 N (Ans)

Q. A body of weight 300 N is lying on a rough horizontal plane having a coefficient of friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.

Soln: Data Given: Weight of the body (W) = 300 N; Coefficient of friction (μ) = 0.3 and angle made by the force with the horizontal (α) = 25°

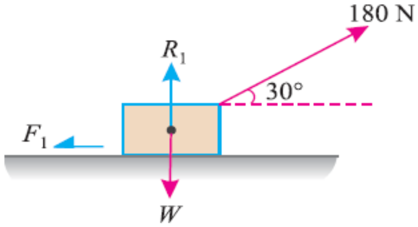


Let P = Magnitude of the force, which can move the body, and F = Force o
Resolving the forces horizontally,
$F = P \cos\alpha = P \cos 25° = P \times 0.9063$ and now resolving the forces verticall
$R = W - P \sin\alpha = 300 - P \sin 25° = 300 - P \times 0.4226$
We know that the force of friction (F),
$0.9063\,P = \mu R = 0.3 \times (300 - 0.4226\,P) = 90 - 0.1268\,P$
$\Rightarrow 90 = 0.9063\,P + 0.1268\,P = 1.0331\,P$

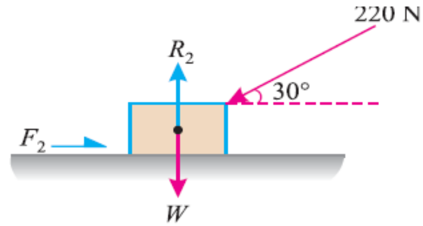
Q. **A body, resting on a rough horizontal plane, required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.**

Data Given: Pull = 180 N; Push = 220 N and angle at which force is inclined with horizontal plane (α) = 30°
Let W = Weight of the body
R = Normal reaction, and
μ = Coefficient of friction.
First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction ($F1$) will act towards left as shown in Fig. a

Resolving the forces horizontally,
$F1 = 180 \cos 30° = 180 \times 0.866 = 155.9$ N and now resolving the forces vertically,
$R1 = W - 180 \sin 30° = W - 180 \times 0.5 = W - 90$ N
We know that the force of friction ($F1$),
$155.9 = \mu R1 = \mu(W - 90)$……………..(i)



(*a*) Pull of 180 N

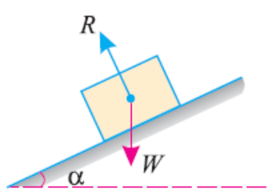


(*b*) Pull of 220 N

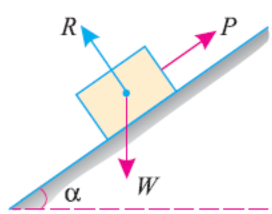
Now consider a push of 220 N acting on the body. We know that in this case, the force of friction ($F2$) will act towards right as shown in Fig. (*b*).
Resolving the forces horizontally,
$F2 = 220 \cos 30° = 220 \times 0.866 = 190.5$ N and now resolving the forces horizontally,
$R2 = W + 220 \sin 30° = W + 220 \times 0.5 = W + 110$ N
We know that the force of friction ($F2$),
$190.5 = \mu R2 = \mu(W + 110)$ ...(*ii*)
Dividing equation (*i*) by (*ii*)
$155.9/190.5=(W -90)/(W+110)$
$155.9\ W + 17\ 149 = 190.5\ W - 17\ 145$
$34.6\ W = 34\ 294$
$W=991.2N$
Now substituting the value of W in equation (*i*),
$155.9 = \mu(991.2 - 90) = 901.2\mu$
$\mu=0.173$

EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE

Consider a body, of weight W, lying on a rough plane inclined at an angle α with the horizontal as shown in Fig. (*a*) and (*b*). If the inclination of the plane, with the horizontal, is less than the angle of friction, the body will be automatically in equilibrium as shown in Fig. (*a*). But, if the inclination of the plane is more than the angle of friction, the body will move down. And an upward force (P) will be required to resist the body from moving down the plane as shown in Fig (*b*).



(a)



(b)

Though there are many types of forces, for the movement of the body, some important forces are
1. Force acting along the inclined plane.

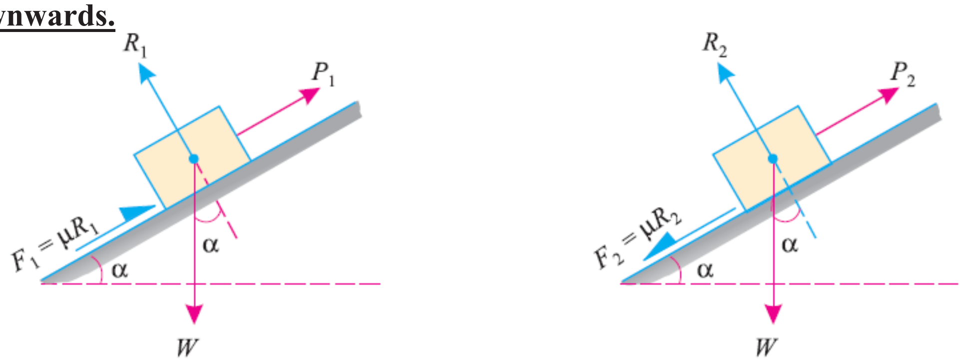
2. Force acting horizontally.

EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING ALONG THE INCLINED PLANE

Consider a body lying on a rough inclined plane subjected force acting along the inclined plane, which keeps it in equilibrium as shown in Fig. (a) and (b). Let W = Weight of the body, α = Angle, which the inclined plane makes with the horizontal, R = Normal reaction, μ = Coefficient of friction between the body and the inclined plane, and φ = Angle of friction, such that μ = tan φ.

Case-1

Minimum force (P1) which will keep the body in equilibrium, when it is at the point of sliding downwards.



(*a*) Body at the point of sliding downwards

(*b*) Body at the point of sliding upwards

In this case, the force of friction (F1 = μ.R1) will act upwards, as the body is at the point of sliding downwards as shown in Fig. (a). Now resolving the forces along the plane,

P1 = W sin α – μ.R1 ...(i)

and now resolving the forces perpendicular to the plane.

R1 = W cos α ...(ii)

Substituting the value of R1 in equation (i), P1 = W sin α – μ W cos α = W (sin α – μ cos α)

and now substituting the value of μ = tan φ in the above equation,

P1 = W (sin α – tan φ cos α)

Multiplying both sides of this equation by cos φ,

P1 cos φ = W (sin α cos φ – sin φ cos α) = W sin (α – φ)

⇨ P1= W sin (α – φ)/Cos φ (Ans)

Case2

Maximum force (P2) which will keep the body in equilibrium, when it is at the point of sliding upwards.

In this case, the force of friction (F2 = μ.R2) will act downwards as the body is at the point of sliding upwards as shown in Fig. (b).

Now resolving the forces along the plane, P2 = W sin α + μ.R2 ...(i)

and now resolving the forces perpendicular to the plane,

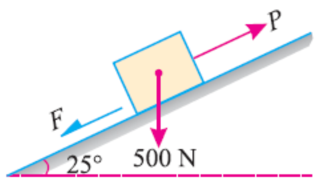
R2 = W cos α ...(ii)

Substituting the value of R2 in equation (i), P2 = W sin α + μ W cos α = W (sin α + μ cos α)

and now substituting the value of μ = tan φ in the above equation,

P2 = W (sin α + tan φ cos α)
Multiplying both sides of this equation by cos φ,
P2 cos φ = W (sin α cos φ + sin φ cos α) = W sin (α + φ)
⇨ P2= W sin (α + φ)/Cos φ (Ans)

Q. A body of weight 500 N is lying on a rough plane inclined at an angle of 25° with the horizontal. It is supported by an effort (P) parallel to the plane as shown in Fig. Determine the minimum and maximum values of P, for which the equilibrium can exist, if the angle of friction is 20°.

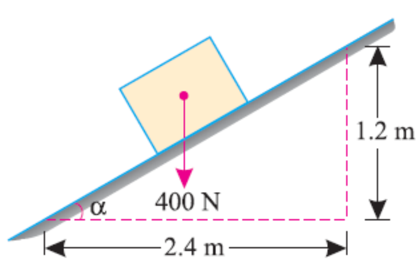


Data Given: Weight of the body (W) = 500 N ; Angle at which plane is inclined (α) = 25° and angle of friction (φ) = 20°.

<u>Minimum value of P</u>
We know that for the minimum value of P, the body is at the point of sliding downwards. We also know that when the body is at the point of sliding downwards, then the force
P1= W sin (α – φ)/Cos φ
⇨ P1= (500x sin (25 – 20))/Cos 20
⇨ P1=46.4N Ans.

<u>Maximum value of P</u>
We know that for the maximum value of P, the body is at the point of sliding upwards. We also know that when the body is at the point of sliding upwards, then the force
P2= W sin (α + φ)/Cos φ
⇨ P2= (500x sin (25 +20))/Cos 20
P2=376.2 N

Q. An inclined plane as shown in Fig. is used to unload slowly a body weighing 400 N from a truck 1.2 m high into the ground. The coefficient of friction between the underside of the body and the plank is 0.3. State whether it is necessary to push the body down the plane or hold it back from sliding down. What minimum force is required parallel to the plane for this purpose?



Data Given: Weight of the body (W) = 400 N and coefficient of friction (μ) = 0.3.
Whether it is necessary to push the body down the plane or hold it back from sliding down.

We know that tan α=1.2/2.4 =0.5 or α = 26.5° and
Normal reaction, R = W cos α = 400 cos 26.5° N = 400 × 0.8949 = 357.9 N
∴ Force of friction, F = μR = 0.3 × 357.9 = 107.3 N ... (i)
Now resolving the 400 N force along the plane = 400 sin α = 400 × sin 26.5° N = 400 × 0.4462 = 178.5 N ...(ii)
We know that as the force along the plane (which is responsible for sliding the body) is more than the force of friction, therefore the body will slide down. Or in other words, it is not necessary to push the body down the plane, rather it is necessary to hold it back from sliding down.
We know that the minimum force required parallel to the plane to hold the body back, P = 178.5 – 107.3 = 71.2 N Ans.

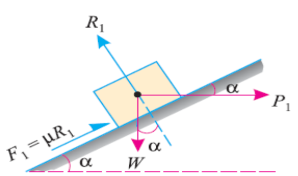
EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING HORIZONTALLY

Consider a body lying on a rough inclined plane subjected to a force acting horizontally, which keeps it in equilibrium as shown in Fig. (a) and (b).
W = Weight of the body
α = Angle, which the inclined plane makes with the horizontal
R = Normal reaction
μ = Coefficient of friction between the body and the inclined plane
And φ = Angle of friction, such that μ = tan φ.

If the force is not there, the body will slide down on the plane. Now we shall discuss the following two cases:

Case-1

Minimum force (P1) which will keep the body in equilibrium, when it is at the point of sliding downwards

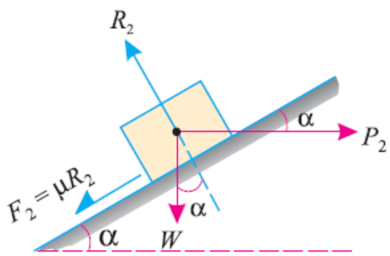


(a) Body at the point of sliding downwards

In this case, the force of friction (F1 = μ.R1) will act upwards, as the body is at the point of sliding downwards as shown in Fig. (a).
Now resolving the forces along the plane, P1 cos α = W sin α – μ R1 ...(i)
and now resolving the forces perpendicular to the plane, R1 = W cos α + P1 sin α ...(ii) Substituting this value of R1 in equation (i),
P1 cos α = W sin α – μ(W cos α + P1 sin α) = W sin α – μW cos α – μP1 sin α
⇨ P1 cos α + μP1 sin α = W sin α – μW cos α
⇨ P1(cos α + μ sin α) = W (sin α – μ cos α)
⇨ P1= W (sin α – μ cos α)/ (cos α + μ sin α)

Case-2

Maximum force (P2) which will keep the body in equilibrium, when it is at the point of sliding upwards



(*b*) Body at the point of sliding upwards

In this case, the force of friction (F2 = μR2) will act downwards, as the body is at the point of sliding upwards as shown in Fig. (b).

Now resolving the forces along the plane, P2 cos α = W sin α + μR2 ...(iii)

and now resolving the forces perpendicular to the plane, R2 = W cos α +P2 sin α ...(iv)

Substituting this value of R2 in the equation (iii),

P2 cos α = W sin α + μ (W cos α + P2 sin α) = W sin α + μ W cos α + μ P2 sin α

⇨ P2 cos α – μ P2 sin α = W sin α + μ W cos α

⇨ P2 (cos α – μ sin α) = W (sin α + μ cos α)

⇨ P2= W (sin α + μ cos α)/ (cos α – μ sin α)

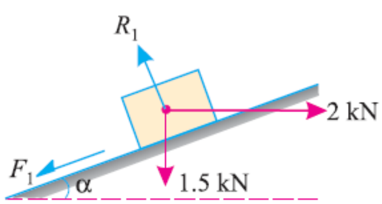
Q. A load of 1.5 KN, resting on an inclined rough plane, can be moved up the plane by a force of 2 kN applied horizontally or by a force 1.25 kN applied parallel to the plane. Find the inclination of the plane and the coefficient of friction.

Data Given: Load (W) = 1.5 kN

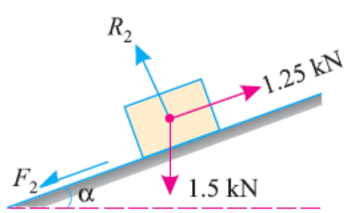
Horizontal effort (P1) = 2 kN

and effort parallel to the inclined plane (P2) = 1.25 kN.

Let α = Inclination of the plane, and φ = Angle of friction.



(*a*) Horizontal force



(*b*) Force parallel to the plane

Consider the load of 1.5 kN subjected to a horizontal force of 2 kN as shown in Fig. (a).

We know that when the force is applied horizontally, then the magnitude of the force, which can move the load up the plane, P = W tan (α + φ)

or 2 = 1.5 tan (α + φ)

∴ tan (α + φ)=2/1.5=1.33

⇨ (α + φ)=53.1^0

Now consider the load of 1.5 KN subjected to a force of 1.25 kN along the plane as shown in Fig. (b).

We Know that when the force is applied parallel to the plane, then the magnitude of the force, which can move the load up the plane,

P= W sin (α + φ)/Cos φ

or 1.25= 1.5 sin 53.1/Cos φ

⇨ φ = 16.3° and α = 53.1° – 16.3° = 36.8°

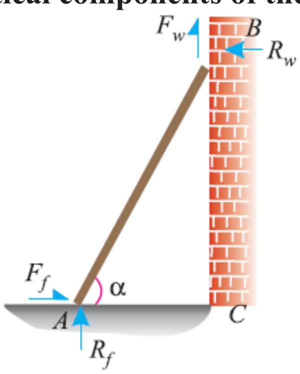
We know that the coefficient of friction, μ = tan φ = tan 16.3° = 0.292 Ans.

LADDER FRICTION

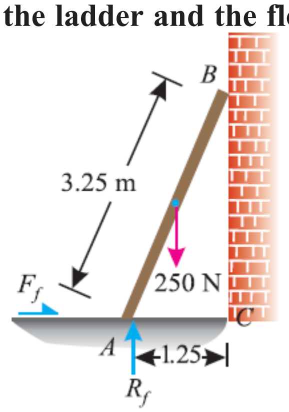
Ladder is a device for climbing or scaling on the roofs or walls. It consists of two long uprights of wood, iron or rope connected by a number of cross pieces called rungs.

As the upper end of the ladder tends to slip downwards, therefore the direction of the force of friction between the ladder and the wall (Fw) will be upwards as shown in the figure. Similarly, as the lower end of the ladder tends to slip away from the wall, therefore the direction of the force of friction between the ladder and the floor (Ff) will be towards the wall as shown in the figure.

Since the system is in equilibrium, therefore the algebraic sum of the horizontal and vertical components of the forces must also be equal to zero.



Q. A uniform ladder of length 3.25 m and weighing 250 N is placed against a smooth vertical wall with its lower end 1.25 m from the wall. The coefficient of friction between the ladder and floor is 0.3. What is the frictional force acting on the ladder at the point of contact between the ladder and the floor? Show that the ladder will remain in equilibrium



in this position.

Data Given: Length of the ladder (l) = 3.25 m

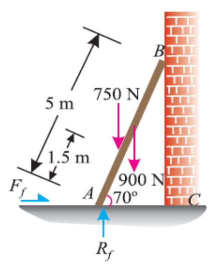
Weight of the ladder (w) = 250 N

Distance between the lower end of ladder and wall = 1.25 m and coefficient of friction between the ladder and floor (μf) = 0.3.

Let Ff = Frictional force acting on the ladder at the point of contact between the ladder and floor,
and Rf = Normal reaction at the floor.
Since the ladder is placed against a smooth vertical wall, therefore there will be no friction at the point of contact between the ladder and wall.
Resolving the forces vertically, Rf = 250 N
From the geometry of the figure, we find that
$BC=\sqrt{(3.25^2-1.25^2)}=3$ m

Taking moments about B and equating the same,
Ff × 3 = (Rf × 1.25) – (250 × 0.625) = (250 × 1.25) – 156.3 = 156.2 N
⇨ Ff=52.1N
We know that the maximum force of friction available at the point of contact between the ladder and the floor = μRf = 0.3 × 250 = 75 N
Thus we see that the amount of the force of friction available at the point of contact (75 N) is more than the force of friction required for equilibrium (52.1 N). Therefore the ladder will remain in an equilibrium position.

Q. **A ladder 5 meters long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750N stands on a rung 1.5 metre from the bottom of the ladder. Calculate the coefficient of friction between the ladder and the floor.**



Data Given: Length of the ladd
Angle which the ladder makes
Weight of the ladder (w1) = 90
Weight of man (w2) = 750 N
and distance between the man and bottom of ladder = 1.5 m.
Forces acting on the ladder are shown in Fig.
Let μf = Coefficient of friction between ladder and floor and
Rf = Normal reaction at the floor.
Resolving the forces vertically,
Rf = 900 + 750 = 1650 N ...(i)
∴Force of friction at A Ff = μf × Rf = μf × 1650 ...(ii)
Now taking moments about B, and equating the same,
Rf × 5 sin 20° = (Ff × 5 cos 20°) + (900 × 2.5 sin 20°) + (750 × 3.5 sin 20°)
Rf=(Ff × 5 cos20°) + (4875 sin 20°) =(μf × 1650 × 5 cos 20°) + 4875 sin 20°
Now substituting the values of Rf and Ff from equations (i) and (ii)
1650 × 5 sin 20° = (μf × 1650 × 5 cos 20°) + (4875 sin 20°)
Dividing both sides by 5 sin 20°, 1650 = (μf × 1650 cot 20°) + 975 =(μf × 1650 × 2.7475) + 975 = 4533 μf + 975

⇨ μf=0.15

CHAPTER-4

CENTRE OF GRAVITY AND MOMENT OF INERTIA

The words centre and gravity are derived from the Latin (or Greek) words "centrum" and "gravitatio". The centre (centroid) represents the centre of mass that is in the cross-section of the diagonals of the body, and gravity – the weight is the attractive force between particles in the universe under which the celestial bodies move.

The **center of gravity (CG)** of an object is the imaginary point on the body where the total weight of the body is assumed to be concentrated or It is the point at which weight of the object is evenly dispersed and all sides are in balance.

CENTROID

Centroid is the geometric centre of the body. The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The centre of area of such figures is known as *centroid*. The method of finding out the centroid of a figure is the same as that of finding out the centre of gravity of a body.

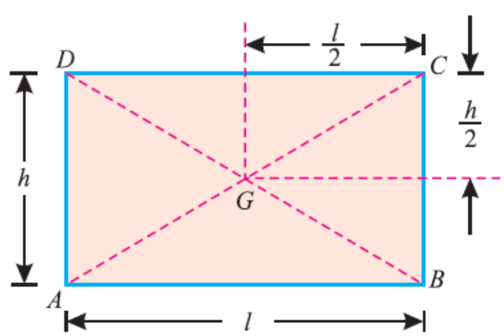
Difference between CG and Centroid

(i) Centre of gravity is the point where the total weight of the body acts while centroid is the geometric centre of the object.

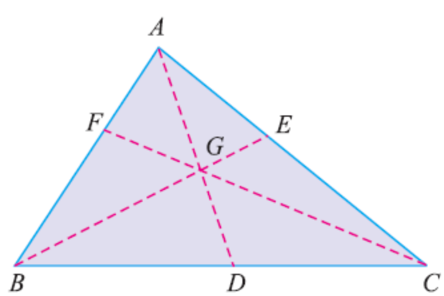
(ii) CG is the centre of mass of a geometric object with any density where as centroid is the centre of mass of a geometric object of uniform density.

(iii) For the objects with uniform density centre of gravity and centroid coincide.

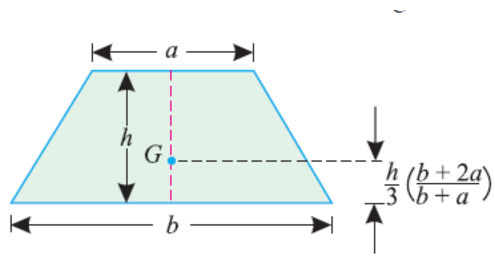
CG of some geometrical cross sections



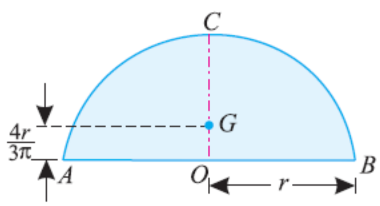
The centre of gravity of a rectangle (or a parallelogram) is at the point, where its diagonals meet each other. It is also a middle point of the length as well as the breadth of the rectangle



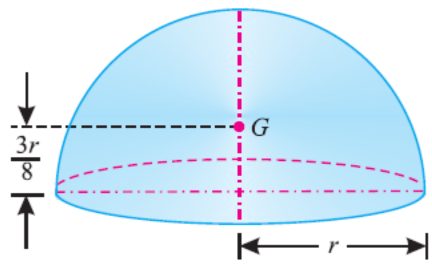
The centre of gravity of a triangle is at the point, where the three medians (a median is a line connecting the vertex and middle point of the opposite side) of the triangle meet



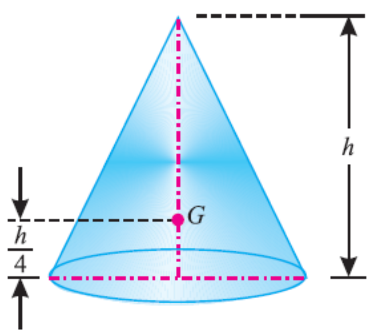
The centre of gravity of a trapezium with parallel sides a and b is at a distance of h/3.((2a+b)/(a+b))measured form the side b



The centre of gravity of a semicircle is at a distance of $\frac{4r}{3\pi}$ from its base measured along the vertical radius



The centre of gravity of a hemisphere is at a distance of $\frac{3r}{8}$ from its base measured along the vertical radius.



The centre of gravity of right circular solid cone is at a distance of h/4 from its base, measured along the vertical axis

The centre of gravity of a cube is at a distance of l/2 from every face (where l is the length of each side).

The centre of gravity of a sphere is at a distance of d/2 from every point (where d is the diameter of the sphere).

CENTRE OF GRAVITY OF PLANE FIGURES

The plane geometrical figures (such as T-section, I-section, L-section etc.) have only areas but no mass. The centre of gravity of such figures is found out in the same way as that of solid bodies. The centre of area of such figures is known as centroid, and coincides with the centre of gravity of the figure.

Let x^- and y^- be the co-ordinates of the centre of gravity with respect to some axis of reference, then

$$x^- = \frac{a1x1 + a2x2 + a3x3 + \dots}{a1 + a2 + a3}$$

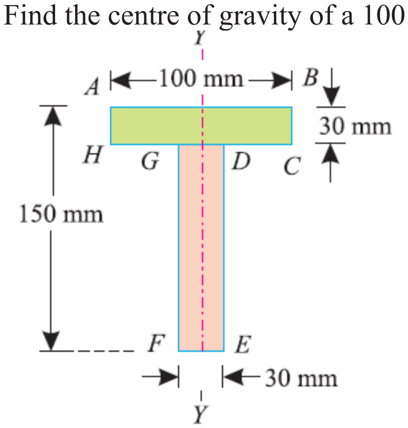
$y^{-}=\frac{a1y1 + a2y2 + a3y3 + \ldots}{a1 + a2 + a3}$

Where a1, a2, a3........ etc., are the areas into which the whole figure is divided.
x1, x2, x3 ..... etc., are the respective co-ordinates of the areas a1, a2, a3....... on X-X axis with respect to same axis of reference.
y1, y2, y3....... etc., are the respective co-ordinates of the areas a1, a2, a3....... on Y-Y axis with respect to same axis of the reference.

CENTRE OF GRAVITY OF SYMMETRICAL SECTIONS

Sometimes, the given section, whose centre of gravity is required to be found out, is symmetrical about X-X axis or Y-Y axis. In such cases, the procedure for calculating the centre of gravity of the body is very much simplified; as we have only to calculate either x^{-} or y^{-}. **This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry.**

Q. Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section.

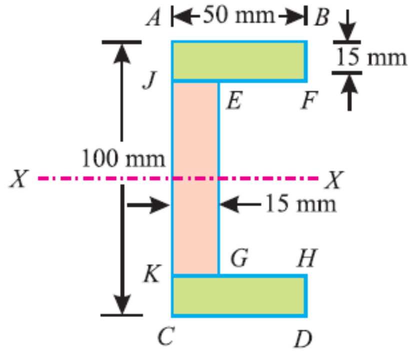


As the section is symmetrical about Y-Y axis, bisecting the web, therefore its centre of gravity will lie on this axis. Split up the section into two rectangles ABCH and DEFG as shown in Fig.
Let bottom of the web FE be the axis of reference.

(i) Rectangle ABCH a1 = 100 × 30 = 3000 mm2 and y1= (150-(30/2))=135mm
(ii) Rectangle DEFG a2 = 120 × 30 = 3600 mm2 and y2=120/2=60mm

$$y^{-}=\frac{a1y1 + a2y2}{a1 + a2} = \frac{3000 \times 135 + 3600 \times 60}{3000 + 3600} = 94.1\text{mm}$$

Q. Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm.



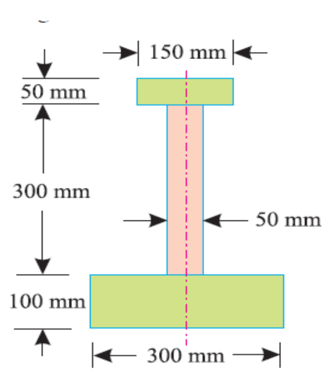
As the section is symmetrical about *X-X* axis, therefore its centre of gravity will lie on this axis. Now split up the whole section into three rectangles *ABFJ*, *EGKJ* and *CDHK* as shown in Fig.

Let the face *AC* be the axis of reference.

(i) Rectangle ABFJ
$a1 = 50 \times 15 = 750$ mm^2
x1=50/2=25mm
(ii) Rectangle EGKJ
$a2 = (100 – 30) \times 15 = 1050$ mm2
X2=15/2=7.5mm
(iii)Rectangle CDHK
$a3 = 50 \times 15 = 750$ mm2
x3=50/2=25mm
We know that distance between the centre of gravity of the section and left face of the section *AC*,

$$\bar{x} = \frac{a1x1 + a2x2 + a3x3}{a1 + a2 + a3} = \frac{750 \times 25 + 1050 \times 7.5 + 750 \times 25}{750 + 1050 + 750} = 17.8\text{mm}$$

Q. An I-section has the following dimensions in mm units :
Bottom flange = 300 × 100
Top flange = 150 × 50
Web = 300 × 50
Determine mathematically the position of centre of gravity of the section.



As the section is symmetrical about Y-Y axis, bisecting the web, therefore its centre of gravity will lie on this axis. Now split up the section into three rectangles as shown in Fig.
Let bottom of the bottom flange be the axis of reference.

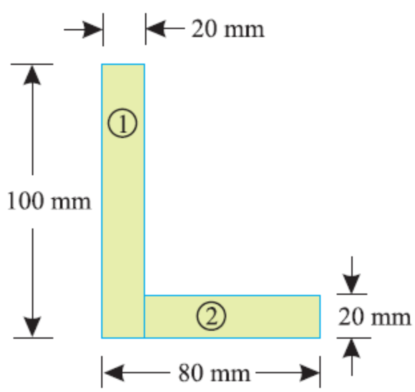
(i) For Bottom flange a1 = 300 × 100 = 30 000 mm2
Y1=100/2=50mm
(ii) For Web a2 = 300 × 50 = 15 000 mm2
Y2=100+(300/2)=250mm
(iii) For Top flange a3 = 150 × 50 = 7500 mm2
Y3=100+300+(50/2)=425mm

We know that distance between centre of gravity of the section and bottom of the flange,

$$\bar{y}=\frac{a1y1+a2y2+a3y3}{a1+a2+a3}=\frac{30000\times 50+15000\times 250+7500\times 425}{30000+15000+7500}=160.7\text{mm}$$

CENTRE OF GRAVITY OF UNSYMMETRICAL SECTIONS

Sometimes, the given section, whose centre of gravity is required to be found out, is not symmetrical either about X-X axis or Y-Y axis. In such cases, we have to find out both the values of $\bar{x}$ and $\bar{y}$.
Q. Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.



Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(i) Rectangle 1 a1 = 100 × 20 = 2000 mm2

X1=20/2=10mm

Y1=100/2=50mm

(ii) Rectangle 2 a2 = (80 – 20) × 20 = 1200 mm2

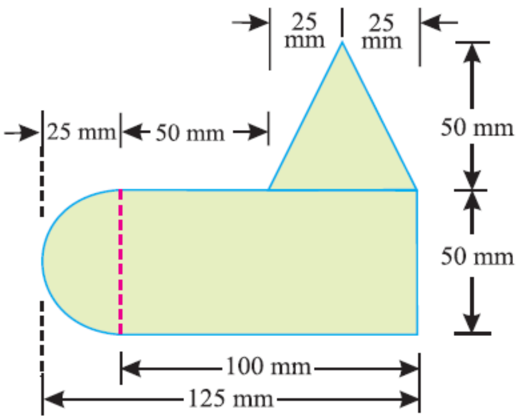
X2=20+(60/2)=50mm

Y2=20/2=10mm

$$\bar{y}=\frac{a1y1+a2y2}{a1+a2}=\frac{2000\times 50+1200\times 10}{2000+1200}=35\text{mm}$$

$\bar{x} = \frac{a1x1 + a2x2}{a1 + a2} = \frac{2000 \times 10 + 1200 \times 50}{2000 + 1200}$ =25mm

Q. A uniform lamina shown in Fig. consists of a rectangle, a circle and a triangle. Determine the centre of gravity of the lamina. All dimensions are in mm.



As the section is not symmetrical about any axis, therefore we have to find out the values of both $\bar{x}$ and $\bar{y}$ for the lamina.

Let left edge of circular portion and bottom face rectangular portion be the axes of reference.

(i) Rectangular portion a1 = 100 × 50 = 5000 mm2
x1=25+(100/2)=75mm
y1=50/2=25mm

(ii) Semicircular portion $a2=(\pi r^2/2)=982mm^2$
x2=25-(4r/3π)=14.4mm
y2=50/2=25mm

(iii) Triangular portion $a3=(50\times50)/2=1250mm^2$
x3 = 25 + 50 + 25 = 100 mm
y3= 50+ (50/3) =66.7mm

We know that distance between centre of gravity of the section and left edge of the circular portion,

$\bar{x} = \frac{a1x1 + a2x2 + a3x3}{a1 + a2 + a3} = \frac{5000 \times 75 + 982 \times 14.4 + 1250 \times 100}{5000 + 982 + 1250}$ =71.1mm

Similarly, distance between centre of gravity of the section and bottom face of the rectangular portion,

$\bar{y} = \frac{a1y1 + a2y2 + a3y3}{a1 + a2 + a3} = \frac{5000 \times 25 + 982 \times 25 + 1250 \times 66.7}{5000 + 982 + 1250}$ = 32.2mm

MOMENT OF INERTIA

Moment of Inertia is a measure of the resistance of a body to angular acceleration about a given axis. Area moment of inertia is known as area moment of moment.

MOMENT OF INERTIA OF A PLANE AREA

Consider a plane area, whose moment of inertia is required to be found out. Split up the whole area into a number of small elements.

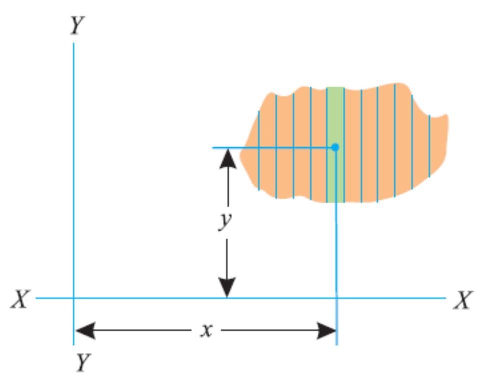
Let a1, a2, a3, ... = Areas of small elements, and r1, r2, r3, ... = Corresponding distances of the elements from the line about which the moment of inertia is required to be found out. Now the moment of inertia of the area $=a1r1^2+a2r2^2+a3r3^2+\ldots\ldots\ldots\ldots= \sum a\, r^2$

Unit of moment of inertia:

m^4 or mm^4

MOMENT OF INERTIA BY INTEGRATION

The moment of inertia of an area may also be found out by the method of integration as discussed below: Consider a plane figure, whose moment of inertia is required to be found out about X-X axis and Y-Y axis as shown in Fig . Let us divide the whole area into a no. of strips. Consider one of these strips. Let dA = Area of the strip x = Distance of the centre of gravity of the strip on X-X axis and y = Distance of the centre of gravity of the strip on Y-Y axis. We know that the moment of inertia of the strip about Y-Y axis = dA . x2 Now the moment of inertia of the whole area may be found out by integrating above equation. i.e., $I_{YY} = \sum dA\ .\ x2$ Similarly $I_{XX} = \sum dA\ .\ y2$



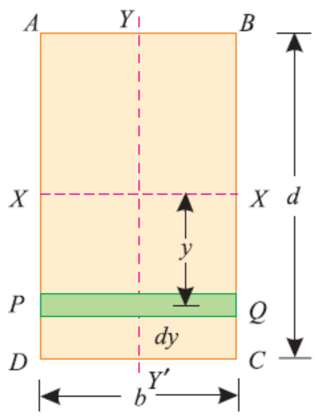
MOMENT OF INERTIA OF A RECTANGULAR SECTION

Consider a rectangular section ABCD as shown in Fig. whose moment of inertia is required to be found out. Let b = Width of the section and d = Depth of the section. Now consider a

strip PQ of thickness dy parallel to X-X axis and at a distance y from it as shown in the figure .

∴ Area of the strip = b.dy

We know that moment of inertia of the strip about X-X axis, = Area × y^2 = (b. dy) y^2 = b. y^2. dy. Now moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina i.e. from (-d/2) to (+d/2)



$$I_{xx}=\int_{-d/2}^{+d/2} by^2 dy = b\int_{-d/2}^{+d/2} y^2 dy = b\left[\frac{y^3}{3}\right]_{-d/2}^{+d/2} = \frac{bd^3}{12}$$

Similarly $I_{yy}=\frac{db^3}{12}$

Q. Find the moment of inertia of a rectangular section 30 mm wide and 40 mm deep about X-X axis and Y-Y axis.

Data Given: Width of the section (b) = 30 mm and depth of the section (d) = 40 mm.

We know that moment of inertia of the section about an axis passing through its centre of gravity and parallel to X-X axis,

$I_{xx}=\frac{bd^3}{12}=\frac{30 \times 40^3}{12}=160\times10^3 mm^4$

Similarly $I_{yy}=\frac{db^3}{12}=\frac{40 \times 30^3}{12}=90\times10^3 mm^4$

Moment of Inertia of the circular section about an axis passing through it's centre

$I_{xx}=\frac{\pi d^4}{64}$ and $I_{yy}=\frac{\pi d^4}{64}$

Moment of inertia of the triangular section about an axis through its centre of gravity and parallel to X-X axis =$\frac{bh^3}{36}$

THEOREM OF PERPENDICULAR AXIS

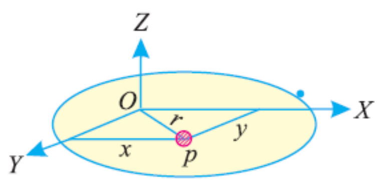
It states, If I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by: $I_{ZZ} = I_{XX} + I_{YY}$

Proof : Consider a small lamina (P) of area da having co-ordinates as x and y along OX and OY two mutually perpendicular axes on a plane section as shown in Fig. . Now consider a plane OZ perpendicular to OX and OY. Let (r) be the distance of the lamina (P) from Z-Z axis such that OP = r. From the geometry of the figure, we find that r2 = x2 + y2

We know that the moment of inertia of the lamina P about X-X axis, I_{XX} = da. y2 [I = Area × (Distance)2]

Similarly, I_{YY} = da. x2 and I_{ZZ} = da. r2 = da (x2 + y2) (as r2 = x2 + y2)

⇨ Izz= da. x2 + da. y2 = $I_{YY} + I_{XX}$



Q**. Find out the polar moment of Inertia of the circle**.

Using perpendicular axis theorem $I_{ZZ}=I_{XX}+I_{YY}=\frac{\pi d^4}{64}+\frac{\pi d^4}{64}=\frac{\pi d^4}{32}$

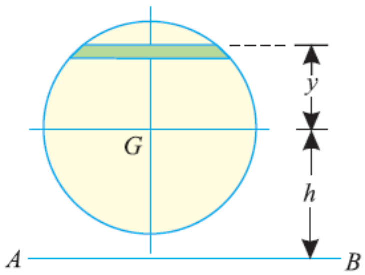
THEOREM OF PARALLEL AXIS

It states, If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I_G, then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by: $I_{AB} = I_G + ah^2$

where I_{AB} = Moment of inertia of the area about an axis AB,

l_G = Moment of Inertia of the area about its centre of gravity

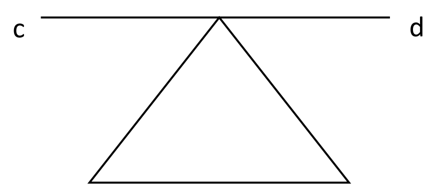
a = Area of the section, and h = Distance between centre of gravity of the section and axis AB.



Proof: Consider a strip of a circle, whose moment of inertia is required to be found out about a line AB as shown in Fig. Let δa = Area of the strip y = Distance of the strip from the centre of gravity the section and h = Distance between centre of gravity of the section and the axis AB. We know that moment of inertia of the whole section about an axis passing through the centre of gravity of the section = δa. y^2 and moment of inertia of the whole section about an axis passing through its centre of gravity, $I_G = \sum \delta a. y^2$

∴ Moment of inertia of the section about the axis AB, $I_{AB} = \sum \delta a\,(h + y)^2 = \sum \delta a\,(h2 + y2 + 2\,h\,y) = (\sum h2.\,\delta a) + (\sum y2.\,\delta a) + (\sum 2\,h\,y\,.\,\delta a) = a\,h2 + I_G + 0$ It may be noted that $\sum h2\,.\,\delta a = a\,h2$ and $\sum y2\,.\,\delta a = I_G$ [as per equation (i) above] and $\sum \delta a.y$ is the algebraic sum of moments of all the areas, about an axis through centre of gravity of the section and is equal to .ay, where y is the distance between the section and the axis passing through the centre of gravity, which is zero.

Q. **Find the moment of inertia of a triangular section about an axis through its vertex and parallel to the base?**



$I_{cd} = I_G + a\,h2 = \frac{bh^3}{36} + \frac{bh}{2} \times (\frac{2h}{3})^2 = bh^3/4$

MOMENT OF INERTIA OF A COMPOSITE SECTION

The moment of inertia of a composite section may be found out by the following steps:

1. First of all, split up the given section into plane areas (*i.e.*, rectangular, triangular, circular etc., and find the centre of gravity of the section).

2. Find the moments of inertia of these areas about their respective centres of gravity.

3. Now transfer these moment of inertia about the required axis (*AB*) by the Theorem of Parallel Axis, *i.e.*,

$I_{AB} = I_G + ah2$

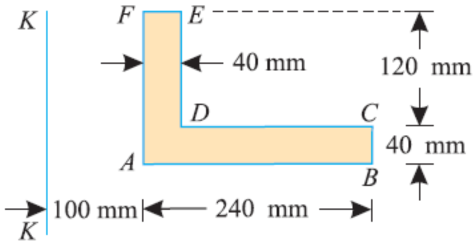
where I_G = Moment of inertia of a section about its centre of gravity and parallel to the axis.

a = Area of the section,

h = Distance between the required axis and centre of gravity of the section.

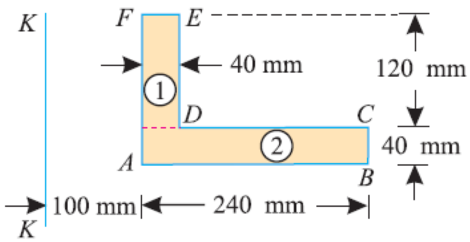
4. The moments of inertia of the given section may now be obtained by the algebraic sum of the moment of inertia about the required axis.

Q. **Compute the moment of inertia of the above area about axis K-K.**



As the moment of inertia is required to be found out about the axis K-K,

Let us split up the area into two rectangles 1 and 2 as shown in Fig.



We know that moment of inertia of section (1) about its centre of gravity and parallel to axis K-K is $I_{G1}=\frac{120 \times 40^3}{12}=640\times10^3 mm^4$

and distance between centre of gravity of section (1) and axis K-K,

h1=100+(40/2)=120mm

Moment of inertia of section (1) about axis K-K = $I_{kk} = I_{G1} + a1h1^2$=640×10^3+((120×40)×120^2)= 69.76×10^6 mm^4

Similarly, moment of inertia of section (2) about its centre of gravity and parallel to axis K-K,

$I_{G2}=\frac{40 \times 240^3}{12}=46.08\times10^6$ mm^4

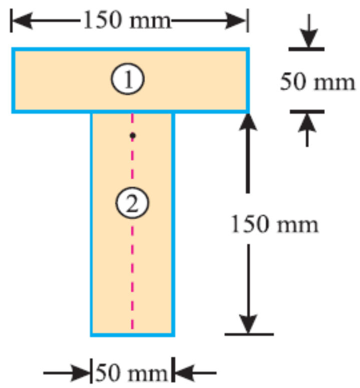
and distance between centre of gravity of section (2) and axis K-K,

h2=100+(240/2)=220mm

Moment of inertia of section (2) about the axis K-K

= $I_{kk} = I_{G2} + a2h2^2=46.08\times10^6+((240\times40)\times220^2)= 510.72\times 10^6$ mm^4

Q. Find the moment of inertia of a T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about X-X and Y-Y axes through the centre of gravity of the section.



The given T-section is shown in Fig.

As the section is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles viz., 1 and 2 as shown in figure. Let bottom of the web be the axis of reference.

(i) Rectangle (1) a1 = 150 × 50 = 7500 mm2

and y1=150+(50/2)=175mm

(ii) Rectangle (2) a2 = 150 × 50 = 7500 mm2   and y2= 150/2=75mm

$\bar{y}=\frac{a1y1+a2y2}{a1+a2}=125\text{mm}$

Moment of inertia about X-X axis We also know that M.I. of rectangle (1) about an axis through its centre of gravity and parallel to X-X axis

$I_{G1}=\frac{150\times 50^3}{12}$

and distance between centre of gravity of rectangle (1) and X-X axis, h1 = 175 – 125 = 50 mm

Moment of inertia of rectangle (1) about X-X axis $I_{G1} + a1h1^2$=20.3125× 10^6 mm^4

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to X-X axis

$I_{G2}=\frac{50\times 150^3}{12}$

and distance between centre of gravity of rectangle (2) and X-X axis, h2 = 125 – 75 = 50 mm ∴ Moment of inertia of rectangle (2) about X-X axis $I_{G2} + a2h2^2$=32.8125× 10^6 mm^4

Now moment of inertia of the whole section about X-X axis, IXX = $(20.3125 \times 10^6) + (32.8125 \times 10^6) = 53.125 \times 10^6$ mm4

Moment of inertia about Y-Y axis

We know that M.I. of rectangle (1) about Y-Y axis=$\frac{50 \times 150^3}{12}$

and moment of inertia of rectangle (2) about Y-Y axis=$\frac{150 \times 50^3}{12}$

Now moment of inertia of the whole section about Y-Y axis, IYY = (14.0625 × 106) + (1.5625 × 106) = 15.625 × 106 mm^4

CHAPTER-5

SIMPLE MACHINES

SIMPLE MACHINE

In a broad sense, a simple machine may be defined as a device, which enables us to do some useful work at some point or to overcome some resistance, when an effort or force is applied to it, at some other convenient point.

Example: Screw, Pulley, Lever, Wedge, Wheel and axle etc.

COMPOUND MACHINE

A compound machine may be defined as a device, consisting of a number of simple machines, which enables us to do some useful work at a faster speed or with a much less effort as compared to a simple machine.

Example: Bicycle, crane, car jack, lawn mover etc.

LIFTING MACHINE

It is a device, which enables us to lift a heavy load (*W*) by applying a comparatively smaller effort (*P*).

INPUT OF A MACHINE

The input of a machine is the work done on the machine. In a lifting machine, it is measured by the product of effort and the distance through which it has moved.

OUTPUT OF A MACHINE

The output of a machine is the actual work done by the machine. In a lifting machine, it is measured by the product of the weight lifted and the distance through which it has been lifted.

EFFICIENCY OF A MACHINE

It is the ratio of output to the input of a machine and is generally expressed as a percentage.

Mathematically, efficiency, $\boldsymbol{\eta = \frac{Output}{Input} \times 100}$

IDEAL MACHINE

If the efficiency of a machine is 100% *i.e.*, if the output is equal to the input, the machine is called as a perfect or an *ideal machine.*

VELOCITY RATIO

The velocity ratio (briefly written as V.R.) is the ratio of distance moved by the effort (y) to the distance moved by the load (x) and is always expressed in pure number. Mathematically, velocity ratio, VR=y/x

MECHANICAL ADVANTAGE

The mechanical advantage (briefly written as M.A.) is the ratio of weight lifted (W) to the effort applied (P) and is always expressed in pure number. Mathematically, mechanical advantage, MA=W/P

RELATION BETWEEN EFFICIENCY, MECHANICAL ADVANTAGE AND VELOCITY RATIO OF A LIFTING MACHINE

Now consider a lifting machine, whose efficiency is required to be found out.
W = Load lifted by the machine,
P = Effort required to lift the load,
Y = Distance moved by the effort, in lifting the load, and
x = Distance moved by the load.
We know that M.A.=W/P and VR=y/x
We also know that
input of a machine= Effort applied × Distance through which the effort has moved= $P \times y$ ...(*i*)
and output of a machine = Load lifted × Distance through which the load has been lifted= $W \times x$....*(ii)*
Efficiency, = Output/Input=Wx/Py=(W/P)/(y/x)=MA/VR

For an ideal machine MA=VR as η=100%

Q. In a certain weight lifting machine, a weight of 1 kN is lifted by an effort of 25 N. While the weight moves up by 100 mm, the point of application of effort moves by 8 m. Find mechanical advantage, velocity ratio and efficiency of the machine.

Data Given: Weight (W) = 1 kN = 1000 N ; Effort (P) = 25 N ; Distance through which the weight is moved (x) = 100 mm = 0.1 m and distance through which effort is moved (y) = 8 m.

Mechanical advantage of the machine.

We know that mechanical advantage of the machine MA=W/P=1000/25=40

Velocity ratio of the machine

We know that velocity ratio of the machine VR=y/x=8/0.1=80

Efficiency of the machine

We also know that efficiency of the machine, **η =MA/VR=40/80=0.5=50%**

REVERSIBILITY OF A MACHINE

If a machine is also capable of doing some work in the reversed direction, after the effort is removed then such a machine is called a reversible machine and its action is known as reversibility of the machine.

CONDITION FOR THE REVERSIBILITY OF A MACHINE

Consider a reversible machine, whose condition for the reversibility is required to be found out.
Let W = Load lifted by the machine,
P = Effort required to lift the load,
y = Distance moved by the effort, and
x = Distance moved by the load.
We know that input of the machine= $P \times y$ ...(*i*) and output of the machine = $W \times x$ ...(*ii*)
We also know that machine friction= Input – Output = $(P \times y) - (W \times x)$ ...(*iii*)
In a reversible machine, the output of the machine should be more than the machine friction, when the effort (P) is zero. *i.e.*, $W \times x > P \times y - W \times x$
or $2\, W \times x > P \times y$

- ⇨ Wx/Py>0.5
- ⇨ (W/P)/(y/x)>0.5
- ⇨ MA/VR>0.5

So η >0.5

Hence the condition for a machine, to be reversible, is that its efficiency should be *more* than 50%.

SELF-LOCKING MACHINE

Sometimes, a machine is not capable of doing any work in the reversed direction, after the effort is removed. Such a machine is called a non-reversible or *self-locking machine.* The condition for a machine to be non-reversible or self-locking is that its efficiency should *not be more than* 50%.

Q. A certain weight lifting machine of velocity ratio 30 can lift a load of 1500N with the help of 125 N effort. Determine if the machine is reversible.

Data Given: Velocity ratio (V.R.) = 30; Load (W) = 1500 N and effort (P) = 125 N.

MA=W/P=1500/125=12

η=MA/VR=12/30=0.4=40%

Since efficiency of the machine is less than 50%, therefore the machine is non-reversible.

Q. In a lifting machine, whose velocity ratio is 50, an effort of 100 N is required to lift a load of 4 kN. Is the machine reversible ? If so, what effort should be applied, so that the machine is at the point of reversing ?

Data Given: Velocity ratio (V.R.) = 50 ; Effort (P) = 100 N and load (W) = 4 kN = 4000 N.
Reversibility of the machine
We know that M.A=W/P=4000/100=40

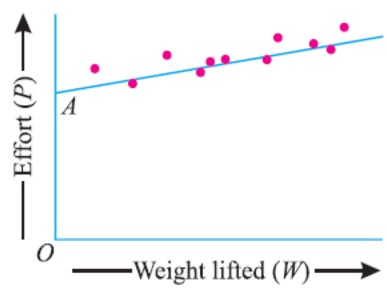
η=MA/VR=40/50=0.8=80%

Since efficiency of the machine is more than 50%, therefore the machine is reversible. **Ans.**
Effort to be applied

A little consideration will show that the machine will be at the point of reversing, when its efficiency is 50% or 0.5.
Let P1 = Effort required to lift a load of 4000 N when the machine is at the point of reversing.
MA=W/P1=4000/P1
0.5=MA/VR=4000/P1/VR=4000/P1/50=80/P1
=> P1=160N

FRICTION IN A MACHINE

Every machine cannot be frictionless. It has been observed that there is always some amount of friction present in every machine, which can be expressed on a graph of effort (*P*) and load or weight lifted (*W*). If we record the various values of efforts required to raise the corresponding loads or weights and plot a graph between effort and load, we shall obtain a straight line *AB* as shown in Fig.



Here the intercept OA represents the amount of friction present in the machine. The machine friction may be expressed either on the effort side or on the load side. **If expressed on the effort side, the friction may be defined as an additional effort required to overcome the frictional froce. But if expressed on the load side, the friction may be defined as the additional load that can be lifted or the additional resistance that can be overcome.**
Let P = Actual effort (considering the machine friction) required to lift a weight
P' = Ideal weight (neglecting the machine friction) required to lift the same weight
W = Actual weight (considering machine friction) lifted by an effort and W' = Ideal weight (neglecting machine friction) lifted by the same effort.
In the above equations (P) is greater than (P'). Similarly, (W') is greater than (W). It is thus obvious, from the above equations, that (P – P') is the amount of effort required to overcome the machine friction and (W' – W) is the load equivalent to the machine friction.
It may be noted, that in the case of an ideal machine, to lift a weight W, the effort required is P' only; whereas to lift a load W' effort required is P. But in the case of an actual machine to lift a weight W, effort required is P. We know that efficiency of the machine, **η=MA/VR=W/P/VR**
In case of an ideal machine, the efficiency is equal to 1. Substituting this value of efficiency equal to 1 equation (i), W/P=VR
As already discussed, to lift a load W, in the case of an ideal machine, the effort required is P'. Now substituting P' instead of P in equation (ii),W/ P'=VR
P'=W/VR

Now the effort (P) is required to lift the load (W), when the friction is considered and the effort (P') is required to lift the same load when the machine is considered to be ideal i.e., when the friction is neglected. P- P'=P-(W/VR)
We know that (P – P') is the friction, if expressed in terms of effort. ∴ **F_{effort}=P-(W/VR)**

Now the effort (P) will lift a load (W') if the mchine friction is neglected and the same effort will lift a load (W) if the machine friction is considered. Therefore substituting W' for W (considering the machine to be ideal, i.e., neglecting the machine friction) in equation (ii), W'/P=VR

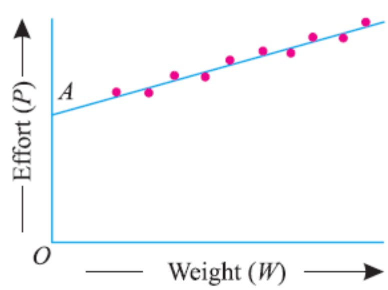
W'=P×VR
W' – W = (P × V.R.) – W
We know that (W' – W) is the friction, if expressed in terms of load.
∴ $F_{(load)}$ = (P × V.R.) – W

Q. In a certain machine, an effort of 100 N is just able to lift a load of 840 N, Calculate efficiency and friction both on effort and load side, if the velocity ratio of the machine is 10.
Given: Effort (P) = 100 N ; Load (W) = 840 N and velocity ratio (V.R.) = 10.
We know that MA=W/P=840/100=8.4
η=MA/VR=8.4/10=84%
Friction of the machine
We know that friction of the machine in terms of effort, F_{effort}=P-(W/VR)=100-(840/10)=16N
and friction of the machine in terms of load,
F(load) = (P × V.R.) – W = (100 × 10) – 840 = 160N
Here an effort of 16 N is required to overcome the friction. Or in other words, this effort can lift an additional load of 160 N

LAW OF A MACHINE

The term 'law of a machine' may be defined as relationship between the effort applied and the load lifted. Thus for any machine, if we record the various efforts required to raise the corresponding loads, and plot a graph between effort and load, we shall get a straight line AB as shown in Fig.

Here the intercept OA represents the amount of friction offered by the machine. Or in other words, this is the effort required by the machine to overcome the friction, before it can lift any load.



Mathematically, the law of a lifting machine is given by the relation : P = mW + C
where P = Effort applied to lift the load
m = A constant (called coefficient of friction) which is equal to the slope of the line AB
W = Load lifted, and C = Another constant, which represents the machine friction, (i.e. OA).

Q. What load can be lifted by an effort of 120 N, if the velocity ratio is 18 and efficiency of the machine at this load is 60% ? Determine the law of the machine, if it is observed that an

effort of 200 N is required to lift a load of 2600 N and find the effort required to run the machine at a load of 3.5 kN.

Data Given: Effort (P) = 120 N ; Velocity ratio (V.R.) = 18 and efficiency (η) = 60% = 0.6.
Let W = Load lifted by the machine.
We know that M.A.=W/P=W/120
0.6=MA/VR=W/120/18
W = 0.6 × 2160 = 1296 N
Law of the machine In the second case, P = 200 N and W = 2600 N Substituting the two values of P and W in the law of the machine, i.e., P = m W + C, 120 = m × 1296 + C ...(i) and 200 = m × 2600 + C ...(ii) Subtracting equation (i) from (ii),
80 = 1304 m
m=0.06
and now substituting the value of m in equation (ii)
200 = (0.06 × 2600) + C = 156 + C

- ⇨ C = 200 – 156 = 44
- ⇨ Now substituting the value of m = 0.06 and C = 44 in the law of the machine, **P = 0.06 W + 44**

Effort required to run the machine at a load of 3.5 kN.
Substituting the value of W = 3.5 kN or 3500 N in the law of machine, P = (0.06 × 3500) + 44 = 254 N

Q. In a lifting machine, an effort of 40 N raised a load of 1 kN. If efficiency of the machine is 0.5, what is its velocity ratio ? If on this machine, an effort of 74 N raised a load of 2 kN, what is now the efficiency ? What will be the effort required to raise a load of 5 kN ?

Data Given: When Effort (P) = 40 N; Load (W) = 1 kN = 1000 N; Efficiency (η) = 0.5; When effort (P) = 74 N and load (W) = 2 kN = 2000 N.
MA=W/P=1000/40=25
0.5=MA/VR=25/VR
VR=50
Efficiency when P is 74 N and W is 2000 N
MA=W/P=2000/74=27
η=27/50=54%
Effort required to raise a load of 5 kN or 5000 N

Substituting the two values of P and W in the law of the machine, i.e. P = mW + C 40 = m × 1000 + C ...(i) and 74 = m × 2000 + C ...(ii)
Subtracting equation (i) from (ii),
34 = 1000 m
m=0.034
and now substituting this value of m in equation (i), 40 = (0.034 × 1000) + C = 34 + C
∴ C = 40 – 34 = 6
Substituting these values of m = 0.034 and C = 6 in the law of machine, P = 0.034 W + 6 ...(iii)
∴ Effort required to raise a load of 5000 N, P = (0.034 × 5000) + 6 = 176 N

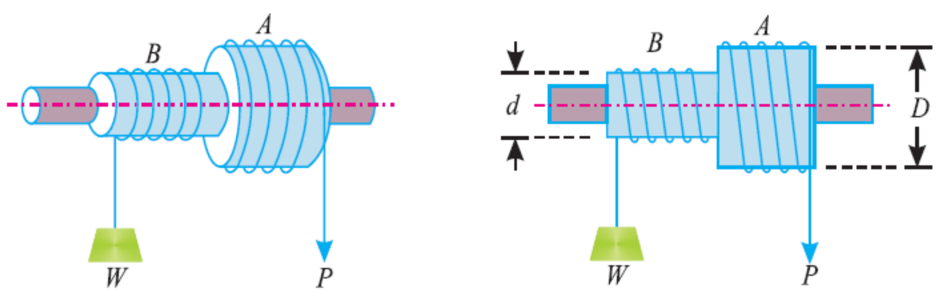
Q. What load will be lifted by an effort of 12 N, if the velocity ratio is 18 and efficiency of the machine at this load is 60 % ? If the machine has a constant friction resistance, determine the law of the machine and find the effort required to run this machine at (i) no load, and (ii) a load of 900 N.

Data Given: Effort (P) = 12 N ; Velocity ratio (V.R.) = 18 and efficiency (η) = 60 % = 0.6
Let W = Load lifted by the machine
MA=W/P=W/12
0.6=MA/VR=W/12/18
W=129.6N
We know that effort lost in friction, Feffort=P-(W/VR)=12-(129.6/18)=4.8N
Since the frictional resistance is constant, therefore 4.8 N is the amount of friction offered by the machine. Now substituting the values of P = 12 and C = 4.8 in the law of the machine we get
12 = m × 129.6 + 4.8 ...(as P = mW + C)
m= 1/18
Law of the machine will be given by the equation
P=(1/18)W+4.8
Effort required to run the machine at no load
Substituting the value of W = 0 in the law of the machine (for no load condition), P = 4.8 N

Effort required to run the machine at a load of 900 N
Substituting the value of W = 900 N in the law of machine P=(1/18)900+4.8=54.8N

SIMPLE WHEEL AND AXLE

A simple wheel and axle is shown in the figure, in which the wheel A and axle B are keyed to the same shaft. The shaft is mounted on ball bearings, order to reduce the frictional resistance to a minimum. A string is wound round the axle B, which carries the load to be lifted. A second string is wound round the wheel A in the opposite direction to that of the string on B.



Let D = Diameter of effort wheel
d = Diameter of the load axle
W = Load lifted and P = Effort applied to lift the load.

One end of the string is fixed to the wheel, while the other is free and the effort is applied to this end. Since the two strings are wound in opposite directions, therefore a downward motion of the effort (P) will raise the load (W). Since the wheel as well as the axle are keyed to the same shaft, therefore when the wheel rotates through one revolution, the axle will also rotate through one revolution.

We know that displacement of the effort in one revolution of effort wheel A, = πD ...(i)
and displacement of the load in one revolution = πd ...(ii)

VR=Distance moved by effort/Distance moved by load=$\frac{\pi D}{\pi d}$=D/d

MA=W/P

η=MA/VR

Q. A simple wheel and axle has wheel and axle of diameters of 300 mm and 30 mm respectively. What is the efficiency of the machine, if it can lift a load of 900 N by an effort of 100 N.

Data Given: Diameter of wheel (D) = 300 mm; Diameter of axle (d) = 30 mm; Load lifted by the machine (W) = 900 N and effort applied to lift the load (P) = 100 N

We know that velocity ratio of the simple wheel and axle=D/d=300/30=10

MA=W/P=900/100=9

η=MA/VR=9/10=0.9=90%

Q. A drum weighing 60 N and holding 420N of water is to be raised from a well by means of wheel and axle. The axle is 100 mm diameter and the wheel is 500 mm diameter. If a force of 120 N has to be applied to the wheel, find (i) mechanical advantage, (ii) velocity ratio and (iii) efficiency of the machine.

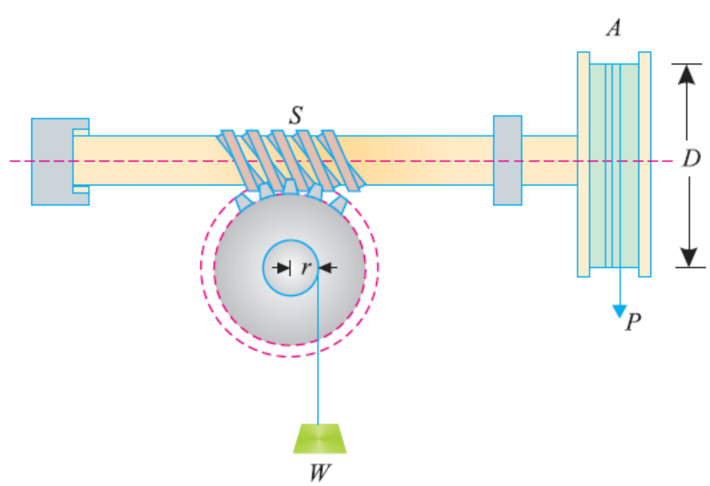
Data Given: Total load to be lifted (W) = 60 + 420 = 480 N; Diameter of the load axle (d) = 100 mm; Diameter of effort wheel (D) = 500 mm and effort (P) = 120 N.

Mechanical advantage=W/P=480/120=4

VR=D/d=500/100=5

η=MA/VR=4/5=0.8=80%

WORM AND WORM WHEEL



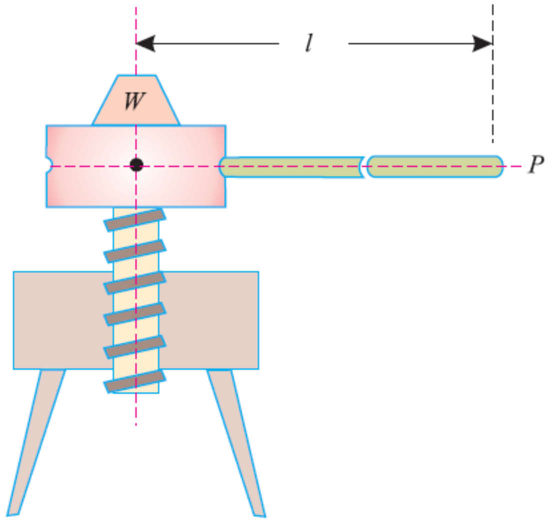
It consists of a square threaded screw, S (known as worm) and a toothed wheel (known as worm wheel) geared with each other, as shown in Fig. A wheel A is attached to the worm, over which passes a rope as shown in the figure. Sometimes, a handle is also fixed to the worm (instead of the wheel). A load drum is securely mounted on the worm wheel. Let D = Diameter of the effort wheel, r = Radius of the load drum W = Load lifted, P = Effort applied to lift the load, and T = No. of teeth on the worm wheel.
We know that distance moved by the effort in one revolution of the wheel (or handle) = πD ...(i)
If the worm is single-threaded (i.e., for one revolution of the wheel A, the screw S pushes the worm wheel through one teeth), then the load drum will move through 1/T revolution
and distance, through which the load will move=$2\pi r/T$
VR= $\pi D/2\pi r/T$
MA=W/P
η=MA/VR

Q. A worm and worm wheel with 40 teeth on the worm wheel has effort wheel of 300 mm diameter and load drum of 100 mm diameter. Find the efficiency of the machine, if it can lift a load of 1800 N with an effort of 24 N.

Data Given: No. of teeth on the worm wheel (T) = 40 ; Diameter of effort wheel = 300 mm Diameter of load drum = 100 mm or radius (r) = 50 mm; Load lifted (W) 1800 N and effort (P) = 24 N. We know that velocity ratio of worm and worm wheel=DT/2r=$(100\times40)/(2\times50)$=120
MA=W/P=1800/24=75
η=MA/VR=75/120=62.5%

SIMPLE SCREW JACK

It consists of a screw, fitted in a nut, which forms the body of the jack. Fig. shows a simple screw jack, which is rotated by the application of an effort at the end of the lever, for lifting the load.
Let l = Length of the effort arm,
p = Pitch of the screw,
W = Load lifted, and
P = Effort applied to lift the load at the end of the lever.
We know that distance moved by the effort in one revolution of screw, = $2\pi l$ ...(i)
and distance moved by the load = p ...(ii)



VR=2πl/P

MA=W/P

η=MA/VR

Q. A screw jack has a thread of 10 mm pitch. What effort applied at the end of a handle 400 mm long will be required to lift a load of 2 kN, if the efficiency at this load is 45%.

Data Given: Pitch of thread (p) = 10 mm; Length of the handle (l) = 400 mm; Load lifted (W) = 2 kN = 2000 N and efficiency (n) = 45% = 0.45.

Let P = Effort required to lift the load.

VR=2πl/P=(2×π×400)/10=251.3

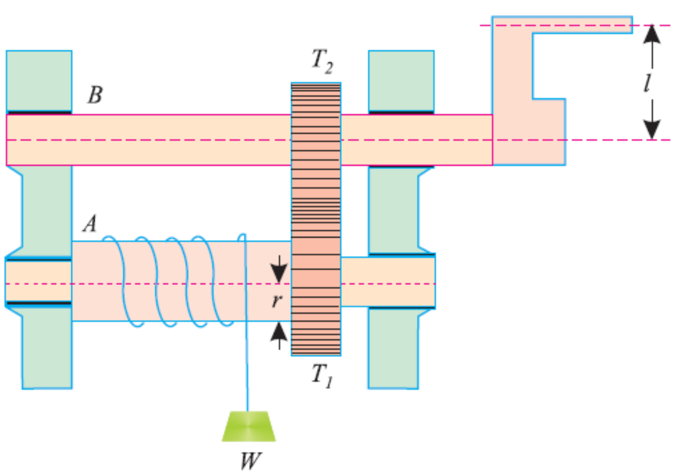
MA=W/P=2000/P

η=MA/VR

0.45=2000/P/251.3

P=17.7N

SINGLE PURCHASE CRAB WINCH



In single purchase crab winch, a rope is fixed to the drum and is wound a few turns round it. The free end of the rope carries the load W. A toothed wheel A is rigidly mounted on the load drum. Another toothed wheel B, called pinion, is geared with the toothed wheel A as shown in Fig.
The effort is applied at the end of the handle to rotate it.
Let T1 = No. of teeth on the main gear (or spur wheel) A
T2 = No. of teeth on the pinion B
l = Length of the handle, r = Radius of the load drum.
W = Load lifted, and P = Effort applied to lift the load.
We know that distance moved by the effort in one revolution of the handle, $=2\pi l$ ...(i)
No. of revolutions made by the pinion B =1 and no. of revolutions made by the wheel A=T2/T1
No. of revolutions made by the load drum=T2/T1
and distance moved by the load=$2\pi r\times(T2/T1)$
$VR=2\pi l/2\pi r\times(T2/T1)=lT1/rT2$
$MA=W/P$
$\eta=MA/VR$

Q. In a single purchase crab winch, the number of teeth on pinion is 25 and that on the spur wheel 100. Radii of the drum and handle are 50 mm and 300 mm respectively. Find the efficiency of the machine and the effect of friction, if an effort of 20 N can lift a load of 300 N.

Data Given: No. of teeth on pinion (T2) = 25; No. of teeth on the spur wheel (T1) = 100; Radius of daum (r) = 50 mm; Radius of the handle or length of the handle (l) = 300 mm; Effort (P) = 20 N and load lifted (W) = 300 N.
Efficiency of the machine
We know that velocity ratio=$2\pi l/2\pi r\times(T2/T1)=lT1/rT2=(300\times100)/(50\times25)=24$
$MA=W/P=300/20=15$
$\eta=MA/VR=15/24=62.5\%$
Effect of friction
We know that effect of friction in terms of load,
F(load) =(P × V.R.) – W = (20 × 24) – 300 = 180 N
and effect of friction in terms of effort Feffort=P-(W/VR)=7.5N

It means that if the machine would have been ideal (i.e. without friction) then it could lift an extra load of 180 N with the same effort of 20 N. Or it could have required 7.5 N less force to lift the same load of 300 N.

CHAPTER-6

DYNAMICS

Kinematics

Kinematics is a subfield of classical mechanics that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without considering the forces that cause them to move.

Kinetics

Kinetics is the branch of classical mechanics that is concerned with the relationship between motion of the body and the force which causes it.

Mass

It is the matter contained in a body.The units of mass are kilogram, tonne etc.

Weight

It is the force, by which the body is attracted towards the centre of the earth. The units of weight are the same as those of force i.e. N, kN etc.

Momentum

It is the quantity of motion possessed by a body. It is expressed mathematically as
Momentum = Mass × Velocity.
The units of momentum depend upon the units of mass and velocity. In S.I. units, the mass is measured in kg, and velocity in m/s, therefore the unit of momentum will be kg-m/s.

Force

It is a very important factor in the field of dynamics also, and may be defined as any cause which produces or tends to produce, stops or tends to stop motion. The units of force, like those of weight, are N, kN etc.

Inertia

It is an inherent property of a body, which offers resistance to the change of its state of rest or uniform motion.

NEWTON'S LAWS OF MOTION

Following are the three laws of motion, which were enunciated by Newton, who is regarded as father of the Science.

Newton's First Law of Motion states that "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force." It is also called the law of inertia. A body at rest has a tendency to remain at rest is called inertia of rest.A body in uniform motion in a straight line has a tendency to preserve its motion. It is called inertia of motion.

Newton's Second Law of Motion states that "The rate of change of momentum is directly proportional to the impressed force, and takes place in the same direction, in which the force acts."

Now consider a body moving in a straight line. Let its velocity be changed while moving.

Let m = Mass of a body,

u = Initial velocity of the body,

v = Final velocity of the body,

a = Constant acceleration,

t = Time, in seconds required to change the velocity from u to v, and

F = Force required to change velocity from u to v in t seconds.

∴ Initial momentum = mu

and final momentum = mv

∴ Rate of change of momentum= (mv-mu)/t=m(v-u)/t=ma

According to Newton's Second Law of Motion, the rate of change of momentum is directly proportional to the the impressed force.

∴ $F \propto ma = kma,$ where k is a constant of proportionality.

For the sake of convenience, the unit of force adopted is such that it produces unit acceleration to a unit mass.

∴ F = ma = Mass × Acceleration.

Q. A body of mass 7.5 kg is moving with a velocity of 1.2 m/s. If a force of 15 N is applied on the body, determine its velocity after 2 s.

Data Given: Mass of body = 7.5 kg ; Velocity (u) = 1.2 m/s ; Force (F) = 15 N and time (t) = 2 s.

We know that acceleration of the body a=F/m=15/7.5=2m/s^2

Velocity of the body after 2 seconds $v = u + at$ = 1.2 + (2 × 2) = 5.2 m/s

Q. vehicle, of mass 500 kg, is moving with a velocity of 25 m/s. A force of 200 N acts on it for 2 minutes. Find the velocity of the vehicle :

(1) when the force acts in the direction of motion, and

(2) when the force acts in the opposite direction of the motion.

Data Given: Mass of vehicle (m) = 500 kg ; Initial velocity (u) = 25 m/s ; Force (F) = 200N and time (t) = 2 min = 120 s

Velocity of vehicle when the force acts in the dirction of motion

We know that acceleration of the vehicle, a=F/m=200/500=0.4m/s^2

Velocity of the vehicle after 120 seconds

v1 = u + at = 25 + (0·4 × 120) = 73 m/s

Velocity of the vehicle when the force acts in the opposite direction of motion.

We know that velcoity of the vehicle in this case after 120 seconds, (when a = – 0.4 m/s2),

v2 = u + at = 25 + (– 0.4 × 120) = –23 m/s

Q. A car of mass 2.5 tonnes moves on a level road under the action of 1 kN propelling force. Find the time taken by the car to increase its velocity from 36 km. p.h. to 54 km.p.h.

Data Given : Mass of the car (m) = 2.5 t ; Propelling force (F) = 1 kN ; Initial velocity (u) = 36 km.p.h. = 10 m/s and final velocity (v) = 54 km.p.h. = 15 m/s
Let t = Time taken by the car to increase its speed.
We know that acceleration of the car, $a=F/m=1/2.5=0.4 m/s^2$
and final velocity of the car (v), $15 = u + at = 10 + 0.4\ t$
$t=12.5$ sec

Q. A man of mass 60 kg dives vertically downwards into a swimming pool from a tower of height 20 m. He was found to go down in water by 2 m and then started rising. Find the average resistance of the water. Neglect the resistance of air.

Data Given : Mass (m) = 60 kg and height of tower (s) = 20 m.

First of all, consider the motion of the man from the top of the tower to the water surface. In this case, initial velocity (u) = 0 (because the man dives) and distance covered (s) = 20 m
Let v = Final velocity of the man when he reaches the water surface.
We know that $v^2 = u^2 + 2gs = (0)^2 + 2 \times 9.8 \times 20 = 392$
$\therefore v = 392 = 19.8$ m/s
Now consider motion of the man from the water surface up to the point in water from where he started rising. In this case, initial velocity (u) = 19.8 m/s ; final velocity (v) = 0 (because the man comes to rest) and distance covered (s) = 2 m

Let a = Retardation due to water resistance.
We know that $v^2 = u^2 + 2as$
$0 = (19.8)^2 - 2a \times 2 = 392 - 4a$ (Minus sign due to retardation)
$a= 98 m/s^2$
and average resistance of the water, $F = ma = 60 \times 98 = 5880$ N

D'ALEMBERT'S PRINCIPLE

It states, "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics."
Force acting on a body.
P = ma, where m = mass of the body, and a = Acceleration of the body.
⇨ $P - ma = 0$

Newton's Third Law of Motion states that "To every action, there is always an equal and opposite reaction."

RECOIL OF GUN

According to Newton's Third Law of Motion, when a bullet is fired from a gun, the opposite reaction of the bullet is known as the recoil of gun.

M = Mass of the gun,

V = Velocity of the gun with which it recoils,

m = mass of the bullet, and

v = Velocity of the bullet after explosion.

∴ Momentum of the bullet after explosion= mv ...(i)

and momentum of the gun = MV ...(ii)

Equating the equations (i) and (ii),

MV = mv

Q. A machine gun of mass 25 kg fires a bullet of mass 30 gram with a velocity of 250 m/s. Find the velocity with which the machine gun will recoil.

Data Given: Mass of the machine gun (M) = 25 kg ; Mass of the bullet (m) = 30 g = 0.03kg and velocity of firing (v) = 250 m/s.

Let V = Velocity with which the machine gun will recoil.

We know that $MV = mv$

$2\,5 \times v = 0.03 \times 250 = 7.5$

v=0.3m/s

Q. A bullet of mass 20 g is fired horizontally with a velocity of 300 m/s, from a gun carried in a carriage; which together with the gun has mass of 100 kg. The resistance to sliding of the carriage over the ice on which it rests is 20 N. Find (a) velocity, with which the gun will recoil, (b) distance, in which it comes to rest, and (c) time taken to do so.

Data Given : Mass of the bullet (m) = 20 g = 0.02 kg ; Velocity of bullet (v) = 300 m/s; Mass of the carriage with gun (M) = 100 kg and resistance to sliding (F) = 20 N

(a) Velocity, with which the gun will recoil

Let V = velocity with which the gun will recoil.

We know that MV = mv

$1\,00 \times V = 0.02 \times 300 = 6$

V=0.06m/s

(b) Distance, in which the gun comes to rest

Now consider motion of the gun. In this case, initial velocity (u) = 0.06 m/s and final velocity

(v) = 0 (because it comes to rest)

Let a = Retardation of the gun, and

s = Distance in which the gun comes to rest.

We know that resisting force to sliding of carriage (F)

20 = Ma = 100 a

$a=0.2 m/s^2$

We also know that $v2 = u2 - 2as$ (Minus sign due to retardation)

$0 = (0.06)2 - 2 \times 0.2\, s = 0.0036 - 0.4\, s$

S=9mm

(c) Time taken by the gun in coming to rest

Let t = Time taken by the gun in coming to rest.
We know that final velocity of the gun (v),
$0 = u + at = 0.06 – 0.2\ t$ ...(Minus sign due to retardation)
t=0.3sec

Equation of motion

u = Initial velocity,
v = Final velocity,
t = Time (in seconds) taken by the particle to change its velocity from u to v.
a = Uniform positive acceleration, and
s = Distance travelled in t seconds.
Since in t seconds, the velocity of the particle has increased steadily from (u) to (v) at the rate of a, therefore total increase in velocity= $a\ t$

1. $v = u + a\ t$
2. $S=ut+(1/2at^2)$
3. $v^2-u^2=2as$

Q. A train travelling at 27 km.p.h is accelerated at the rate of 0.5 m/s2. What is the distance travelled by the train in 12 seconds ?

Data Given : Initial velocity (u) = 27 km.p.h. = 7.5 m/s ; Accceleration (a) = 0.5 m/s2 and time taken (t) = 12 s.
We know that distance travelled by the train, $S=ut+(1/2at^2)=7.5\times12+(0.5\times0.5\times12^2)=126$m

Q. A scooter starts from rest and moves with a constant acceleration of 1.2 m/s2. Determine its velocity, after it has travelled for 60 meters.

Data Given: Initial velocity (u) = 0 (because it starts from rest) Acceleration (a) =1.2 m/s2 and distance travelled (s) = 60 m.
Let v = Final velocity of the scooter.
We know that $v^2 = u^2 + 2as = (0)^2 + 2 \times 1.2 \times 60 = 144$
v= 12 m/s

MOTION UNDER FORCE OF GRAVITY

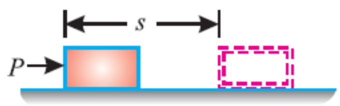
1. $v = u + g\ t$ ...(i)
2. $S=ut+(1/2gt^2)$
3. $v^2-u^2=2gs$

Where g=acceleration due to gravity=9.81m/s^2

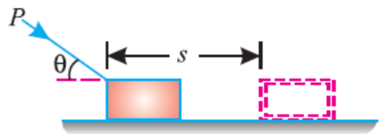
Work

Whenever a force acts on a body, and the body undergoes some displacement, then work is said to be done. A force P, acting on a body, causes it to move through a distance s as shown in Fig.
Then work done by the force P
= Force × Distance

$= P \times s$



(*a*) Body moving in the direction of force



(*b*) Body not moving in the direction of force

Here W=PCosΘ×s

UNITS OF WORK

N-m. It is the work done by a force of 1 N, when it displaces the body through 1 m. It is called joule (briefly written as J), mathematically 1 joule = 1 N-m

kN-m. It is the work done by a force of 1 kN, when it displaces the body through 1 m. It is also called kilojoule (briefly written as kJ). Mathematically 1 kilo-joule = 1 kN-m

Q. A horse pulling a cart exerts a steady horizontal pull of 300 N and walks at the rate of 4·5 km.p.h. How much work is done by the horse in 5 minutes ?

Data Given: Pull (i.e. force) = 300 N ; Velocity (v) = 4·5 km.p.h. = 75 m/ min and time (t) = 5 min.
We know that distance travelled in 5 minutes s = 75 × 5 = 375 m and work done by the horse,
W = Force × Distance = 300 × 375 = 112 500 N-m = 112.5 kN-m = 112.5 kJ

POWER

The power may be defined as the rate of doing work. It is thus the measure of performance of engines. e.g. an engine doing a certain amount of work, in one second, will be twice as powerful as an engine doing the same amount of work in two seconds.

UNITS OF POWER

In S.I. units, the unit of power is watt (briefly written as W) which is equal to 1 N-m/s or 1 J/s. Generally, a bigger unit of power (kW) is used, which is equal to 103 W. Sometimes, a still bigger unit of power (MW) is also used, which is equal to 106 W.

Q. A motor boat is moving with a steady speed of 10 m/s. If the water resistance to the motion of the boat is 600 N, determine the power of the boat engine.

Data Given: Speed of motor boat = 10 m/s and resistance = 600 N
We know that work done by the boat engine in one second = Resistance × Distance = 600 × 10 = 6000 N-m/s = 6 kN-m/s = 6 kJ/s
∴ Power = 6 kW

Q. A locomotive draws a train of mass 400 tonnes, including its own mass, on a level ground with a uniform acceleration, until it acquires a velocity of 54 km.p.h in 5 minutes. If the frictional resistance is 40 newtons per tonne of mass and the air resistance varies with the square of the velocity, find the power of the engine. Take air resistance as 500 newtons at 18 km.p.h.

Data Given : Mass of the locomotive i.e. mass of the train + own mass (m) = 400 t ;
Velocity acquired (v) = 54 km.p.h. = 15 m/s ;
Time (t) = 5 min = 300 s and frictional resistance = 40 N/t = 40 × 400 = 16 000 N = 16 kN
Let a = Acceleration of the locomotive train.
We know that final velocity of the locomotive after 300 seconds (v) 15 = 0 + a × 300 = 300 a

⇨ $a=0.05 m/s^2$

Force required for this acceleration = ma = 400 × 0·05 = 20 kN ...(i)
As the air resistance varies with the square of the velocity, therefore air resistance at 54 km.p.h.
$=500(54/18)^2= 4500$ N= 4·5 kN
Total resistance = 16 + 20 + 4·5 = 40·5 kN and work done in one second = Total resistance × Distance = 40·5 × 15 = 607·5 kN-m/s = 607·5 kJ/s
∴ Power = 607·5 kW

ENERGY

The energy may be defined as the capacity to do work. It exists in many forms i.e., mechanical, electrical chemical, heat, light etc.

UNITS OF ENERGY

As energy is the capacity to do work, therefore the units of energy will be the same as those of the work.

i.e, J, KJ

MECHANICAL ENERGY

Though there are many types of mechanical energies, yet the following two types are important from the subject point of view.

1. Potential energy. 2. Kinetic energy.

POTENTIAL ENERGY

It is the energy possessed by a body, for doing work, by virtue of its position. e.g.,

1. A body, raised to some height above the ground level, possesses some potential energy, because it can do some work by falling on the earth's surface.

2. Compressed air also possesses potential energy, because it can do some work in expanding, to the volume it would occupy at atmospheric pressure.
3. A compressed spring also possesses potential energy, because it can do some work in recovering to its original shape.

Now consider a body of mass (m) raised through a height (h) above the datum level. We know that work done in raising the body = Weight × Distance = (mg) h = mgh
This work (equal to m.g.h) is stored in the body as potential energy.

KINETIC ENERGY

It is the energy, possessed by a body, for doing work by virtue of its mass and velocity of motion.

Now consider a body, which has been brought to rest by a uniform retardation due to the applied force.
Let m = Mass of the body
u = Initial velocity of the body
P = Force applied on the body to bring it to rest,
a = Constant retardation,
and s = Distance travelled by the body before coming to rest.
Since the body is brought to rest, therefore its final velocity, v = 0 and work done, W = Force × Distance = P × s ...(i)
Now substituting value of (P = m.a) in equation (i), W = ma × s = mas ...(ii)
We know that $v^2 = u^2 - 2\,as$ (Minus sign due to retardation)
$\therefore 2as = u^2$ ...(as v = 0)
$as = u^2/2$
Now substituting the value of (a.s) in equation (ii) and replacing work done with kinetic energy
$KE = mu^2/2$

Momentum

Momentum is the product of the mass and velocity of an object. It is a vector quantity, possessing a magnitude and a direction. If *m* is an object's mass and **v** is its velocity (also a vector quantity), then p=mv. In SI units, momentum is measured in kilogram meters per second (kg·m/s). Momentum refers to the quantity of motion an object has.

Impulse

Impulse is a term that quantifies the overall effect of a force acting over time. It is conventionally given the symbol J and expressed in Newton-seconds.
For a constant force, $J = F \cdot \Delta t$. Impulse is the rate of change of momentum of an object.

Collision

This property of bodies, by virtue of which, they rebound, after impact, is called **elasticity**. It may be noted that a body, which rebounds to a greater height is said to be more elastic, than that which rebounds to a lesser height. But, if a body does not rebound at all, after its impact, it is called an **inelastic** body.

PHENOMENON OF COLLISION

Whenever two elastic bodies collide with each other, the phenomenon of collision takes place as given below:
1. The bodies, immediately after collision, come momentarily to rest.
2. The two bodies tend to compress each other, so long as they are compressed to the maximum value.

3. The two bodies attempt to regain its original shape due to their elasticity. This process of regaining the original shape is called restitution.

The time taken by the two bodies in compression, after the instant of collision, is called the time of compression and time for which restitution takes place is called the time of restitution. The sum of the two times of collision and restitution is called time of collision, period of collision, or period of impact.

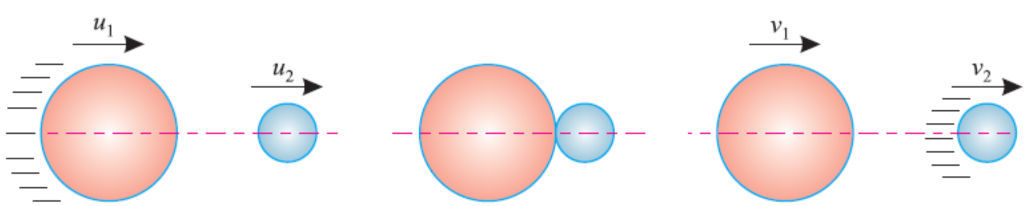
LAW OF CONSERVATION OF MOMENTUM

It states, "The total momentum of two bodies remains constant after their collision or any other mutual action."
Mathematically
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
where m_1 = Mass of the first body,
u_1 = Initial velocity of the first body,
v_1 = Final velocity of the first body, and
m_2, u_2, v_2 = Corresponding values for the second body.

NEWTON'S LAW OF COLLISION OF ELASTIC BODIES

It states, "When two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach." Mathematically,
$(v_2 – v_1) = e (u_1 – u_2)$
where v_1 = Final velocity of the first body,
u_1 = Initial velocity of the first body,
v_2, u_2 = Corresponding values for the second body, and
e = Constant of proportionality.

COEFFICIENT OF RESTITUTION



(a) Before impact (b) During impact (c) After impact

Consider two bodies A and B having a direct impact as shown in Fig.
Let u_1 = Initial velocity of the first body,
v_1 = Final velocity of the first body, and
u_2, v_2 = Corresponding values for the second body.
A little consideration will show, that the impact will take place only if u_1 is greater than u_2. Therefore, the velocity of approach will be equal to $(u_1 – u_2)$. After impact, the separation of the two bodies will take place, only if v_2 is greater than v_1. Therefore the velocity of separation will be equal to $(v_2 – v_1)$. As per Newton's Law of Collision of Elastic Bodies :
Velocity of separation = e × Velocity of approach

$(v_2 - v_1) = e\,(u_1 - u_2)$

where e is a constant of proportionality, and is called the coefficient of restitution. Its value lies between 0 and 1. It may be noted that if e = 0, the two bodies are inelastic. But if e = 1, the two bodies are perfectly elastic.

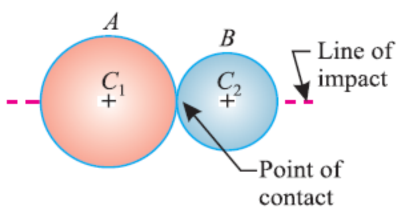
1. If the two bodies are moving in the same direction, before or after impact, then the velocity of approach or separation is the difference of their velocities. But if the two bodies are moving in the opposite directions, then the velocity of approach or separation is the algebraic sum of their velocities.

TYPES OF COLLISIONS

When two bodies collide with one another, they are said to have an impact. Following are the two types of impacts.

1. Direct impact, and 2. Indirect (or oblique) impact.

Direct Impact



The line of impact, of the two colliding bodies, is the lien joining the centres of these bodies and passes through the point of contact or point of collision. If the two bodies, before impact, are moving along the line of impact, the collision is called as direct impact.

Now consider the two bodies A and B having a direct impact as shown in Fig.

Let m_1 = Mass of the first body, u_1 = Initial velocity of the first body,

m_2, u_2, v_2 = Corresponding values for the second body.

From the conservation of momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Q. A ball of mass 1 kg moving with a velocity of 2 m/s impinges directly on a ball of mass 2 kg at rest. The first ball, after impinging, comes to rest. Find the velocity of the second ball after the impact and the coefficient of restitution.

Data Given: Mass of first ball (m_1) = 1 kg ; Initial velocity of first ball (u_1) = 2 m/s ;

Mass of second ball (m_2) = 2 kg; Initial velocity of second ball (u_2) = 0 (because it is at rest) and final velocity of first ball after impact (v_1) = 0 (because, it comes to rest)

Velocity of the second ball after impact.

Let v_2 = Velocity of the second ball after impact.

We know from the law of conservation of momentum that $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

⇨ $(1 \times 2) + (2 \times 0) = (1 \times 0) + (2 \times v_2)$

$\therefore 2 = 2v_2$ or $v_2 = 1$ m/s

Coefficient of restitution

Let e = Coefficient of restitution.
We also know from the law of collision of elastic bodies that (v2 – v1) = e (u1 – u2)

- ⇨ (1 – 0) = e (2 – 0)
- ⇨ E=0.5

Q. A ball overtakes another ball of twice its own mass and moving with 1/7 of its own velocity. If coefficient of restitution between the two balls is 0.75, show that the first ball will come to rest after impact.

Data Given: Mass of first ball (m1) = M kg ; Mass of second ball (m2) = 2 M ; Initial velocity of first ball (u1) = U ; Initial velocity of second ball (u2)=U/7 and coefficient of restitution (e) = 0.75
Let v1 = Velocity of the first ball after impact, and v2 = Velocity of the second ball after impact.
We know from the law of conservation of momentum that m1u1 + m2u2 = m1v1 + m2v2
MU+(2MU/7)=Mv1+2Mv2

$$\frac{9MU}{7} = Mv_1 + 2Mv_2 \quad \text{or} \quad \frac{9U}{7} = v_1 + 2v_2 \qquad ...(i)$$

We also know from the law of collision of elastic bodies that

$$(v_2 - v_1) = e\,(u_1 - u_2) = 0.75\left(U - \frac{U}{7}\right) = \frac{9U}{14}$$

or

$$v_2 = \frac{9U}{14} + v_1$$

Substituting this value of v_2 in equation (i),

$$\frac{9U}{7} = v_1 + 2\left(\frac{9U}{14} + v_1\right) = 3v_1 + \frac{9U}{7} \quad \text{or} \quad v_1 = 0$$

Thus the first ball will come to rest after impact.

Q. The masses of two balls are in the ratio of 2 : 1 and their velocities are in the ratio of 1 : 2, but in the opposite direction before impact. If the coefficient of restitution be 5/6, prove that after the impact, each ball will move back with 5/6th of its original velocity.

Data Given : Mass of first ball (m1) = 2 M ; Mass of second ball (M2) = M ; Initial velocity of first ball (u1) = U ; Initial velocity of second ball (u2) = – 2U (Minus sign due to opposite direction) and coefficient of restitution (e)=5/6

Let v1 = Final velocity of the first ball, and v2 = Final velocity of the second ball.
We know from the law of conservation of momentum that m1u1 + m2u2 = m1v1 + m2v2

- ⇨ 2M × U + M (– 2U) = 2Mv1 + Mv2 or 0 = 2Mv1 + Mv2

  ∴ v2 = – 2v1 ...(i)

We also know from the law of collision of elastic bodies that (v2 – v1) = e (u1 – u2)=5/6[U-(-2U)]=5U/2

Substituting the value of v_2 from equation (i)

$$\left[-2v_1 - (v_1)\right] = \frac{5U}{2} \quad \text{or} \quad v_1 = -\frac{5}{6} \times U$$

Minus sign indicates that the direction of v_1 is opposite to that of U. Thus the first ball will move back with $\frac{5}{6}$th of its original velocity. **Ans.**

Now substituting the value of v_1 in equation (i),

$$v_2 = -2\left(-\frac{5}{6} \times U\right) = +\frac{5}{6} \times 2U$$

Plus sign indicates that the direction of v_2 is the same as that of v_1 or opposite to that of u_2. Thus the second ball will also move back with $\frac{5}{6}$th of its original velocity. **Ans.**

LOSS OF KINETIC ENERGY DURING COLLISION

The kinetic energy may be broadly defined as the energy possessed by a body by virtue of its mass and velocity. Mathematically kinetic energy, $E=1/2mv^2$

where m = Mass of the body, and v = Velocity of the body,

The loss of kinetic energy, during impact, may be obtained by finding out the kinetic energy of the two bodies before and after the impact.

The difference between the kinetic energies of the system, gives the required loss of kinetic energy during impact.

Consider two bodies A and B having a direct impact.

Let m1 = Mass of the first body, u1 = Initial velocity of the first body, v1 = Final velocity of the first body, m2, u2, v2 = Corresponding values for the second body, e = Coefficient of restitution.

We know that kinetic energy of the first body, before impact=$1/2m1u1^2$

and kinetic energy of the second body, before impact=$1/2m2u2^2$

Total kinetic energy of the two bodies, before impact, =$1/2m1u1^2+1/2m2u2^2$

Similarly, total kinetic energy of two bodies, after impact=$1/2m1v1^2+1/2m2v2^2$

Loss of kinetic energy, during impact

EL = E1 – E2= $(1/2m1u1^2+1/2m2u2^2)-(1/2m1v1^2+1/2m2v2^2)$

Q. A ball impinges directly on a similar ball at rest. The first ball is reduced to rest by the impact. Find the coefficient of restitution, if half of the initial kinetic energy is lost by impact.

Data Given : Initial velocity of second body (u2) = 0 (because it is at rest) and final velocity of the first body (v1) = 0 (because it comes to rest by the impact)

Let m1 = Mass of the first body, m2 = m1 = Mass of the second body, (as both the balls are similar)

u1 = Initial velocity of the first body, v2 = Final velocity of the second body, and e = Coefficient of restitution.

We know that kinetic energy of the system before impact, =$1/2m1u1^2+1/2m2u2^2=1/2m1u1^2$

And total kinetic energy of two bodies, after impact=$1/2m1v1^2+1/2m2v2^2=1/2m2v2^2$

Loss of kinetic energy during impact EL = E1 – E2=$1/2m1u1^2-1/2m2v2^2$

Since half of the initial K.E. is equal to loss of K.E. by impact, therefore$1/2(1/2m1u1^2)=1/2m1u1^2-1/2m2v2^2$

⇨ $u1^2=2v2^2$ ……………….(i)

We know from the law of conservation of elastic bodies that (v2 – v1) = e (u1 – u2)

v2 – 0 = e (u1 – 0) (as v1 = 0 and u2 = 0)

∴ v2 = eu1 ...(ii) Substituting the value of v2 in equation (i),$u1^2=2(eu1)^2$

⇨ e=0.707